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Bakerian Lecture: The Effects of Temperature and Pressure on the Thermal Conductivities of Solids. Part II. The Effects of Low Temperatures on the Thermal and Electrical Conductivities of Certain Approximately Pure Metals and Alloys

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XI. BAKERIAN LECTURE.—*The Effects of Temperature and Pressure on the Thermal Conductivities of Solids.—Part II.** *The Effects of Low Temperatures on the Thermal and Electrical Conductivities of Certain Approximately Pure Metals and Alloys.*

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[PLATES 30–31.]

DURING the last fifty years a considerable amount of attention has been bestowed on the question of the variations of the thermal and electrical conductivities of metals with the temperature, but the results obtained by different observers, especially of thermal conductivities, differed so widely from each other that the answer to the thermal part of the question long remained doubtful.† In recent years, however, there has been an accumulation of evidence in favour of a slight decrease of thermal conductivity with increase of temperature from 0° C. to 100° C. in the case of most of the metals. The experiments of LORENZ,‡ and more especially the careful work of JÄGER and DIESSELHORST,§ have contributed greatly to this result. In both these cases the experiments were limited to the range of temperature between 0° C. and 100° C., and it seemed advisable, in view of the importance of both questions in the electronic theories of conduction of heat and electricity in metals, to extend the range over which the theories could be tested, particularly in the direction of low temperatures, where the experiments of DEWAR and FLEMING‖ had already furnished information as to the electrical conductivities. The present paper contains an account of the measurements of the thermal and electrical conductivities carried out for this purpose,

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* Part I., "The Effect of Temperature on the Thermal Conductivities of some Electrical Insulators," appeared in the 'Philosophical Transactions,' A, vol. 204, pp. 433–466 (1905).

† A good account of the subject will be found in GRAETZ'S article in the new edition (1906) of WINKELMANN'S 'Handbuch der Physik,' Band III.

‡ L. LORENZ, 'Ann. der Physik,' 13, p. 422 (1881).

§ W. JÄGER and H. DIESSELHORST, 'Abh. d. Phys.-Techn. Reichsanstalt,' 3, p. 269 (1900).

‖ J. DEWAR and J. A. FLEMING, 'Phil. Mag.,' 36, p. 271 (1893).

and the results obtained. The first section deals with measurements of the thermal, the second with measurements of the electrical, conductivities of certain metals and alloys, and the third section compares the results with the electronic theories.

SECTION I.—THE THERMAL CONDUCTIVITY MEASUREMENTS.

Outline of Method Used.

When the thermal conductivity of a substance which conducts heat readily has to be determined, it is advisable to give the material the form of a wire or thin rod and to allow the heat to flow along the axis. By this arrangement it is easy to insure that the difference of temperature to be measured is not too small, and it has been a favourite one with observers. Sometimes the rod has been exposed to the air of the room, as in the original experiments of BIOT,* sometimes enclosed in a vessel as in those of WIEDEMANN and FRANZ.† As the present experiments were to be carried out at temperatures down to that of liquid air, the former method could not be used without entailing an excessive loss of heat from the sides of the rods. The method of enclosure was therefore adopted.

The scale of the apparatus was mainly determined by the necessity of lowering its temperature to that of liquid air and maintaining it at that temperature. This leads to a reduction in the size of the rods, and must be compensated by an increase in the accuracy of the heat and temperature measurements. This was secured by communicating the heat to one end of the rod and measuring it electrically, and by determining the difference of temperature at two points of the rod by means of two platinum thermometers.

The Apparatus.

The metal rods used, R, fig. 1, were circular cylinders 7 to 8 centims. long, ·585 centim. in diameter, and were accurately turned out of larger rods. In use they were placed vertically, the lower end fitting into a copper disc, D, of 2·69 centims. diameter, 1·2 centims. thick in the centre, 1 centim. thick at the circumference, which in its turn fitted accurately into the lower end of a copper cylinder, T, of 2·69 centims. internal, 3·32 centims. external, diameter, 9·5 centims. long, closed at the top.

On the rod three thin brass sleeves,‡ A, B, and C, 0·66 centim. long, were placed, the fit being of such a nature that the sleeves could be slid along the rod with the fingers without the application of more than a small force.§

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* J. B. BIOT, 'Traité de physique' (1816).

† G. WIEDEMANN and R. FRANZ, 'Ann. der Phys.,' 89, p. 497 (1853).

‡ I had not succeeded at this date in making thin mica rings which would stand the wear of use with the whole of the rods, see p. 428.

§ For the accurate turning and fitting of the rods and sleeves I am indebted to Mr. E. T. COOK, the Mechanician of the University of Manchester.

The outer surface of each ring was covered with shellac varnish, thinly dusted with fine marble powder, and the ring then allowed to dry. On the top of this insulating layer, in the case of the sleeve C, a No. 40 single silk-covered platinoid wire, 30·6 centims. long, was wound, so that the winding occupied the middle portion of the sleeve, leaving a margin of 0·13 centim. at each end. On the insulating layer of each of the two sleeves A and B an equal length of single silk-covered No. 40 pure platinum wire* was similarly wound. The ends of the fine platinoid wire were soldered to the ends of two 55-centim. lengths, L, of No. 22½ double silk-covered and shellac-varnished copper wire, and the ends of the fine platinum wire to two 48-centim. lengths, M or N, of the same copper wire. To obviate stress on the fine wires, the copper wires of each sleeve were placed in parallel grooves in the edge of a thin disc of wood, W, attached to the sleeve by a few turns of silk thread, *s*, wound round the sleeve, wood and wires. In each case a total length of 1·5 centims. of the thin wire used was required to make connections to the ends of the copper leads, the remaining length, 29·1 centims., being actually wound on the sleeve.

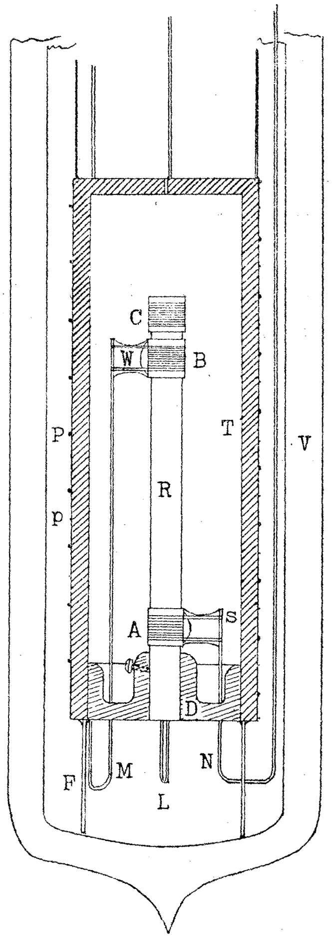

Fig. 1.

The three sets of copper leads passed out of the copper enclosure through three holes in the thin part of the copper disc, which formed the bottom of the enclosure. At the holes extra insulation was provided for the wires, and they could be fixed in position with respect to the disc by three small wedges of wood placed in the holes of the disc.

The copper enclosure was supported on a wire frame, F, by means of which it could be placed in a straight Dewar tube, V, of 4·2 centims. internal diameter, 25 centims. internal depth, so that the bottom of the enclosure was 2 centims. above the bottom of the tube.

The copper leads were brought up to the mercury switch arrangements described on p. 384, by means of which the difference of the resistances of the two platinum coils and the actual resistance of one of them could be found. The current supplied to the heating coil, and the EMF at its terminals, were measured as described on p. 384.

Around the outside of the copper tube an insulated platinoid wire, *p*, having the same resistance as that of the fine platinoid wire of the heating coil, C, was wound, and whenever the current was switched off the heating coil on the rod, it was switched on to that of the tube, so that the amount of heat supplied to the apparatus should be the same throughout an experiment. In addition, a further coil, P, was wound on

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* Obtained specially from Messrs. Johnson, Matthey & Co.

the outside of the tube to allow the temperature of the tube, if necessary, to be rapidly raised.

The Resistance Bridge.

The mercury switch to which the leads from the platinum resistance coils were brought was so arranged that when the difference of resistance of the coils was to be found, they formed two of the arms of a resistance bridge, the other two arms of which consisted of two coils, of about 2 ohms each, of manganin wire wound together and adjusted to equality to within 1 part in 10,000. A dial resistance with mercury contacts, allowing a variation of resistance from 0 to 0·10 ohm in steps of 0·01 ohm, could be placed in series with either of the platinum resistance coils in the bridge, and provided the adjustment necessary for balancing the bridge approximately. The resistance required to give an accurate balance was calculated from the deflections of the galvanometer for the two values of the dial resistance nearest to the balance value. As the leads to the two platinum coils were made as nearly as possible equal to each other, the resistance so found is the difference between the resistances of the two coils. Similarly, when the resistance of one of the coils was to be determined, that coil was, by means of the mercury switch, made one arm of a resistance bridge of which the two equal manganin coils formed two other arms, and the fourth consisted of a resistance box giving 10 ohms in steps of 0·1 ohm, and the 0·01-ohm dial in series. The leads again being as nearly as possible equal, the resistance of the fourth arm, when the bridge was balanced, was equal to the resistance of the platinum coil.

The current used in balancing the bridge was derived from a Leclanché cell connected to the bridge through a resistance of 53 ohms. The connections of the cell could be reversed if necessary.

The galvanometer was of the moving-coil type, and had a resistance of 22 ohms. It could be used either with or without a shunt of 10 to 50 ohms, as occasion required.

The bridge key was a thermo-electric one which, in its normal position, disconnects the cell from the bridge, but leaves the galvanometer connected, and, when depressed, disconnects the galvanometer for an instant, connects the cell, then re-connects the galvanometer. By this means thermo-electric effects in the bridge have no influence on the observations.

Method of Experimenting.

In setting up the apparatus for an experiment, the rod, sleeves, and edges of the copper disc forming the base of the tube were smeared with olive oil, and the joints between rod and base of tube, between rod and sleeves, and between the base of the tube and the tube itself, made in such a way that the oil excluded all air. Good thermal contacts were thus secured. None of these contacts come in the direct line of flow of heat within the region of measurement, but they influence certain small corrections, to be considered presently. Before the rod was placed finally in the tube the leading wires from the coils were wedged in the holes at the base, and the distances apart of the three sleeves measured to 0·01 centim. The apparatus when fitted together was lowered into the Dewar tube, the connections made, and after a few minutes, during which the temperature throughout became uniform, the differences, if any, between the resistances of the platinum wires were measured.

An electric current was then sent round the heating coil on the rod, and was adjusted so that a suitable difference of resistance between the upper and lower platinum coils was obtained. The heating current was then switched from rod to tube, and after 5 to 10 minutes the difference of resistance of the platinum coils, and the actual resistance of the lower coil, were found.

If a second observation, taken 3 to 5 minutes later, gave a different value for the difference of resistance, a further interval of 3 to 5 minutes was allowed and the difference again tested, and this was repeated till the difference of resistance became constant. The current supplied to the heating coil and the E.M.F. at its terminals were then read, and the current switched from tube to rod.

After 5 to 10 minutes the difference of resistance of the platinum coils was again found, and observations taken till it remained constant. The resistance of the lower coil was then measured, and the current in the heating coil and the E.M.F. at its terminals read; then the current was again switched from rod to tube and the measurements repeated.

The mean of the two differences of resistance found when the heating current flowed round the tube, subtracted from the difference found when the heating current flowed round the rod, gives, so long as the rate of rise of temperature of the apparatus is regular, the difference of resistance which would be produced if the heating current were supplied to the rod and the temperature of the tube were kept constant (see p. 397).

The Dewar tube was then filled with liquid air and, when the apparatus had cooled down to the temperature of liquid air, the remaining liquid was poured out by tilting the tube.

Observations of difference of resistance between the platinum coils, with the heating current round the tube, were then made as before till the difference became steady, the resistance of one coil was determined, and the heating current and voltage measured. The current was then switched from tube to rod, and the observations repeated. In this way observations with the heating current alternately round rod and tube were made throughout the gradual rise of temperature of the apparatus from that of liquid air to some temperature above that of the air of the room, the rate of rise of temperature being, if necessary, increased by a constant electric current sent through the supplementary heating coil wound on the copper tube.

Theory of the Apparatus.

If a thin rod of uniform cross section is entirely surrounded by a vessel kept at constant temperature, and has heat supplied to it near one end, while the other is in good thermal contact with the wall of the vessel, the distribution of temperature within the rod, from its point of contact with the vessel to that at which the heat is supplied, will be represented by the equation

$$v = \frac{Q}{(phqk)^{1/2}} \sinh \left(\frac{ph}{qk}\right)^{1/2} x \bigg/ \cosh \left(\frac{ph}{qk}\right)^{1/2} x_C, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v is the excess of temperature of the rod at the cross section situated x centims. from the end of the rod in contact with the vessel, over that of the vessel, p is the perimeter, and q the area of cross section of the rod, h the heat lost per second from 1 sq. centim. of the surface of the rod when its temperature exceeds that of the enclosing vessel by 1° C., k is the thermal conductivity of the rod, and Q is the amount of heat which crosses the section of the rod at any point x_C per second.

In calculating Q from the energy supplied and the heat lost from the surface of the bar beyond x_C, we may, if the point x_C is near the free end of the rod, and the heat is supplied to the rod at a uniform rate between x_C and the free end, take the mean temperature of the surface of the bar between x_C and the end to be identical with the temperature $v_{C'}$ which would be observed at $x_{C'}$, a point which divides the distance between x_C and the end, in the ratio 1 : 2 if the above equation for v held throughout the heated portion of the rod.* If s is the area of the surface of the bar beyond x_C, and H is the total heat supplied to the bar,

$$Q = H - shv_{C'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$v = H \sinh \left(\frac{ph}{qk}\right)^{1/2} x \bigg/ \left[(phqk)^{1/2} \cosh \left(\frac{ph}{qk}\right)^{1/2} x_C + hs \sinh \left(\frac{ph}{qk}\right)^{1/2} x_{C'} \right]. \quad . \quad . \quad (3)$$

If the temperature excesses v_A and v_B at two sections x_A and x_B are observed, then

$$\frac{H}{v_B - v_A} = (phqk)^{1/2} \cosh \left(\frac{ph}{qk}\right)^{1/2} x_C + hs \sinh \left(\frac{ph}{qk}\right)^{1/2} x_{C'} \bigg/ \left[\sinh \left(\frac{ph}{qk}\right)^{1/2} x_B - \sinh \left(\frac{ph}{qk}\right)^{1/2} x_A \right].$$

Hence

$$\frac{H}{v_B - v_A} \left[x_B \sinh \left(\frac{ph}{qk}\right)^{1/2} x_B \bigg/ \left(\frac{ph}{qk}\right)^{1/2} x_B \quad - x_A \sinh \left(\frac{ph}{qk}\right)^{1/2} x_A \bigg/ \left(\frac{ph}{qk}\right)^{1/2} x_A \right]$$
$$= qk \cosh \left(\frac{ph}{qk}\right)^{1/2} x_C + hsx_{C'} \sinh \left(\frac{ph}{qk}\right)^{1/2} x_{C'} \bigg/ \left(\frac{ph}{qk}\right)^{1/2} x_{C'}$$

and

$$k = \left[\frac{H}{v_B - v_A} (x_B S_B - x_A S_A) - hsx_{C'} S_{C'} \right] \bigg/ q \cosh \left(\frac{ph}{qk}\right)^{1/2} x_C, \quad . \quad . \quad . \quad . \quad (4)$$

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* This ratio is exact if the loss of heat from one square centimetre of the surface of the bar may be neglected in comparison with the heat generated per square centimetre.

where S_A, &c., are written for the functions $\sinh (ph/qk)^{1/2} x_A / (ph/qk)^{1/2} x_A$, &c., whose values for small values of the denominators do not differ much from 1.

From this equation k may be calculated if an approximate value of it is known and is substituted in the hyperbolic functions which involve it. In the experiments to be described these functions have values not differing greatly from unity. A sufficiently close approximation to the value of k for this substitution may, in fact, be found by putting

$$\sinh (ph/qk)^{1/2} x \Big/ \left(\frac{ph}{qk}\right)^{1/2} x = \cosh \left(\frac{ph}{qk}\right)^{1/2} x = 1.$$

The effect of temperature on the dimensions of the rods is neglected, as it does not appear likely to influence the value of the conductivity found by so much as 1 part in 300.

In the actual apparatus the heat was supplied to the rod by the passage of the electric current through a platinoid wire wound on an independent sleeve placed on the bar, and the temperatures were measured by the resistances of platinum wires wound on similar sleeves.* It is therefore necessary to determine the effect of the sleeves on the result previously given.

Effect of the Sleeves.

Let v be the excess of temperature of the rod and v' that of the sleeve at points in a plane perpendicular to the axis of the rod, distant x from the central transverse section of the sleeve, the isothermal surfaces being assumed, as in the previous calculation, to be planes perpendicular to the axis.†

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* See note, p. 382.

† The distribution of temperature v throughout a rod of length $2l$, radius R, and conductivity k, which receives heat at a uniform rate h per square centimetre per second through a strip of breadth b of its curved surface at one end, and loses it at a corresponding strip at the other end and at no other point, is given by the equation

$$v = \frac{8hl}{k\pi^2} \sum_{m=0}^{\infty} (-1)^m \frac{1}{(2m+1)^2} \sin (2m+1) \frac{\pi b}{2l} \sin (2m+1) \frac{\pi x}{2l} \, I_0\left(\overline{2m+1} \, \frac{\pi r}{2l}\right) \Big/ I_1\left(\overline{2m+1} \, \frac{\pi R}{2l}\right),$$

where x is the distance of a point from the central transverse section of the rod, r its distance from the axis, and I_0 and I_1 Bessel functions for unreal arguments.

The mean temperature $\bar{v}$ over a strip of the surface of the rod of breadth b, whose centre is a from the central transverse section, is given by

$$\bar{v} = \frac{16Hl^2}{k\pi^4 b^2 R} \Sigma (-1)^m \frac{1}{(2m+1)^3} \sin (2m+1) \frac{\pi b}{2l} \sin (2m+1) \frac{\pi b}{4l} \sin (2m+1) \frac{\pi a}{2l} \, I_0\left(\overline{2m+1} \, \frac{\pi R}{2l}\right) \Big/ I_1\left(\overline{2m+1} \, \frac{\pi R}{2l}\right),$$

where $H = h 2\pi b R$ is the total flow of heat.

If the heat were generated at a uniform rate throughout the material of the rod within the distance b of the end, the flow of heat would be linear and the temperature at a would be given by

$$v = \frac{16Hl^2}{k\pi^4 b R^2} \Sigma (-1)^m \frac{1}{(2m+1)^3} \sin (2m+1) \frac{\pi b}{2l} \sin (2m+1) \frac{\pi a}{2l}.$$

A comparison of the numerical values of the first few terms of the two series, in the case of the shortest rod used, shows that even in that case the error committed in assuming the isothermal surfaces in the rod to be plane is not so large as 1 in 500.

Let q, q' be the areas of cross section of rod and sleeve respectively, p, p' their outer perimeters, and k, k' the conductivities of their materials.

The thickness t'' and conductivity k'' of the thin layer of olive oil between rod and sleeve are small enough compared to those of the rod and sleeve to allow the heat conducted through the layer parallel to the axis of the rod to be neglected. The heat conducted across a length dx of the layer from rod to sleeve in 1 second is equal to $pk''(v-v')\,dx/t''$.

We have then the following equations for the steady distribution of temperature in rod and sleeve :—

In rod

$$qk\frac{d^2v}{dx^2}-\frac{pk''}{t''}(v-v')=0. \qquad (5)$$

In sleeve

$$q'k'\frac{d^2v'}{dx^2}+\frac{pk''}{t''}(v-v')-p'hv'=0. \qquad (6)$$

The last term of the second of these equations is small and may be neglected.*

Taking the temperature at the central transverse section to be zero, we have as the solutions of equations (5) and (6)

$$(qk+q'k')\,v=\mathrm{H}_0\left(x+\frac{q'k'}{qk}\frac{1}{\alpha}\frac{\sinh\alpha x}{\cosh\alpha b}\right), \qquad (7)$$

$$(qk+q'k')\,v'=\mathrm{H}_0\left(x-\frac{q'k'}{qk}\frac{1}{\alpha}\frac{\sinh\alpha x}{\cosh\alpha b}\right), \qquad (8)$$

where H_0 is the flow of heat through the central transverse section of rod and sleeve, and

$$\alpha^2=\frac{pk''}{t''}\left(\frac{1}{qk}+\frac{1}{q'k'}\right).$$

If b' = the length of free rod equivalent in thermal resistance to the length b of rod enclosed by sleeve, we have

$$b'=b\frac{qk}{qk+q'k'}\left\{1+\frac{q'k'}{qk}\frac{\tanh\alpha b}{\alpha b}\right\}. \qquad (9)$$

To determine whether any further simplification is possible, we note that in the apparatus used the constants have the following values :—

b = 0·33 centim., q = 0·268 sq. centim., qk = 0·011 to 0·27 according to the rod used, q' = 0·088 sq. centim., k' = 0·17 to 0·27 according to the temperature, $q'k'$ = 0·015 to 0·024, k'' = 0·0004, t'' = 0·0012 centim., and p = 1·84 centims.

Hence αb lies between 3·2 and 1·7.

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* The error introduced by neglecting h in estimating b', the length of free rod equivalent in thermal resistance to the length b of rod enclosed by sleeve, is about 1 per cent. in the worst case, *i.e.*, that of the shortest rod and lowest conductor. The error produced in the determination of k is about ·1 per cent.

Thus tanh αb will vary between the limits 0·997 and 0·936, having the former value in the case of the worst conducting rods at low temperatures, and the latter in the case of good conducting rods at high temperatures. The error in the estimate of b', if we write tanh $\alpha b = 1$, will even in the latter case only amount to 3 per cent., and the error in k due to it will not exceed 0·04 per cent.

We have, then, the simplified equation

$$b' = \frac{qk}{qk+q'k'}\,b + \frac{q'k'}{qk+q'k'}\,\frac{1}{\alpha},$$

or

$$b' = \frac{qk}{qk+q'k'}\,b + \frac{q'k'}{qk+q'k'}\left[\frac{pk''}{t''}\left(\frac{1}{qk}+\frac{1}{q'k'}\right)\right]^{-1/2}$$

$$= \frac{qk}{qk+q'k'}\,b + \left(\frac{q'k'}{qk+q'k'}\right)^{3/2}\left(\frac{t''}{pk''}\,qk\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The first term on the right is obviously the correction due to the increase of cross section at the sleeve, and the second that due to the ends of the sleeve, at which the flow does not immediately take advantage of the increase of section.

We have seen that in the above equation k varies from ·04 to 1·0 according to the rod under test, while k' varies from 0·17 at 100° to 0·27 at 300° absolute. Hence qk varies from 0·011 to 0·27 according to the rod, and $q'k'$ from 0·015 at 100° to 0·024 at 300° absolute.

The variation of k' with temperature has, however, only a very small effect on the value of b', as will be seen from the following table of values for $k' = 0·17$ and 0·27.

Values of b' in centimetres.

k.	For $k'=17$.	For $k'=27$.	For mean k'.	k.	For $k'=17$.	For $k'=27$.	For mean k'.
·03	·176	·161	·168	·17	·282	·272	·277
·05	·213	·195	·204	·22	·291	·283	·287
·07	·235	·219	·227	·27	·297	·290	·294
·09	·250	·235	·242	·50	·312	·308	·310
·11	·261	·249	·255	1·00	·321	·318	·320
·14	·274	·262	·268				

If for simplicity of calculation we take the mean value of b' as holding throughout the whole range of temperature for any single bar, we make an error in the calculation of k which will not exceed 0·4 per cent. even in the case of the worst conductor tested, Lipowitz's alloy, and will be insignificant for most of the other materials.

Thus we have the following values for x_A, x_B and x_C of the formula (4) for k:—

$x_A = a_1+b'$, where a_1 = distance from enclosure to near edge of first sleeve.

$x_B = x_A+2b'+a_2 = a_1+a_2+3b'$, where a_2 = distance between the near edges of first and second sleeves.

$x_C = x_B+b'+a_3 = a_1+a_2+a_3+4b'$, where a_3 = distance between the near edges of second and third sleeves.

$x_{C'} = x_C+\frac{2}{3}b' = a_1+a_2+a_3+4\cdot7b'$.

a_1 and a_3 were in every case made 0·10 centim.

Estimation of the Small Difference of Temperature between the Rod and the Platinum Resistance Coils.

In addition to a slight difference between the temperature v_0 of the rod and that of the sleeve, v'_0, at the central transverse section of the sleeve, there is a further slight difference between the latter and that of the wire of the platinum resistance v''_0, which may both be calculated with sufficient accuracy for the present purpose by taking

$$pk''\frac{v_0-v'_0}{t''} = p'k'''\frac{v'_0-v''_0}{t'''} = p''hv''_0,$$

where $p = 1\cdot84$ centims. is the perimeter, $t'' = 0\cdot0012$ centim. the thickness, $k'' = 0\cdot0004$ the conductivity, of the layer of oil between the rod and sleeve, $p' = 2\cdot14$ centims. the perimeter, $t''' = 0\cdot014$ centim. the mean thickness,* $k''' = 0\cdot0006$ the conductivity, of the layer of silk and shellac between sleeve and wire, and $p'' = 2\cdot28$ centims. the perimeter of the outer surface of the wire.

From the above we obtain

$$v_0 = (1+29h)\,v''_0.$$

Within the range of temperature covered by the experiments h varies from 0·00016 at 110° absolute to 0·00027 at 300° absolute. Taking the mean value 0·00022 as sufficient for the present purpose, we find

$$v_0 = 1\cdot006v''_0. \qquad (11)$$

In the work which follows, the mean temperature of the platinum coil wound on the sleeve so as to cover 0·4 centim. of its length is taken as identical with the temperature at the centre of its length, the difference between the two being too small to influence the results appreciably.

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* Obtained from the minimum thickness, 0·012 centim., by the addition of a correction suggested by MAXWELL'S 'Electricity and Magnetism,' I., p. 280. The thermal conductivities are only known roughly, so that the final value of the correcting term is not likely to be very accurate. Its influence on the values of the conductivities is, however, very small.

Corrections due to the Leads to the Platinum Resistance Coils.

It has been stated (p. 383) that only 29·1 centims. of the total length, 30·6 centims., of platinum wire were actually wound on the sleeve; the remainder, 1·5 centims., served to make connection to the No. $22\frac{1}{2}$ double silk-covered copper wires leading to the resistance bridge.

Owing to its comparatively large cross section the whole of the copper wire may be assumed to have the temperature of the enclosure, but the temperature in each 0·8 centim. of platinum wire will vary from that of the enclosure to that of the coil of which it is the continuation. This will make the mean temperature of the wire as determined by its resistance slightly less than the temperature of that part of it on the sleeve, and will also lead to the conduction of a small amount of heat from the rod to the leads. We have therefore to determine the magnitude of these effects.

If we measure x along the platinum wire from its junction with the copper wire, we have for the temperature excess v at x the equation

$$v = v''_0 \frac{\sinh \alpha x}{\sinh \alpha l},$$

where v''_0 is, as before, the temperature of the wire on the sleeve, and $\alpha = \left(\frac{ph}{qk}\right)^{1/2}$, where p is the perimeter, q the cross section, of the wire, k the conductivity of its material, and h the external conductivity.

The mean value $\bar{v}$ of v is given by

$$\bar{v} = v''_0 \frac{\cosh \alpha l - 1}{\alpha l \sinh \alpha l} = v''_0 \frac{\sinh \frac{1}{2}\alpha l}{\alpha l \cosh \frac{1}{2}\alpha l} = \frac{v''_0}{2} \frac{\sinh \frac{1}{2}\alpha l}{\frac{1}{2}\alpha l} \Big/ \cosh \tfrac{1}{2}\alpha l.$$

For the platinum wire used, p measured over the silk $= 0·046$ centim., $h = 0·00022$ (mean), $q = 0·00012$ sq. centim., $k = 0·166$; therefore $\alpha = \left(\frac{101}{199}\right)^{1/2} = 0·71$, and $l = 0·8$ centim. Hence

$$\tfrac{1}{2}\alpha l = 0·28 \quad \text{and} \quad \bar{v} = \frac{v''_0}{2} \frac{1·013}{1·039} = 0·487 v''_0.$$

The resistance of the wire will therefore be the same as if, at each end of it, the length on the sleeve were increased by $l' = 0·487 l = 0·365$ centim. and the projecting ends removed. The effective length of the wire as used is, therefore, not the total length, 30·6 centims., but a shorter length, 29·83 centims. Hence

temperature excess of wire on sleeve $= 30·6/29·83 = 1·026$ times mean temperature excess of whole wire.

Hence, in the original equation for k ((4), p. 386), we must take

$$(v_{\text{A}}, v_{\text{B}}) = (1+29h)(v''_{\text{A}}, v''_{\text{B}})$$

and

$$(v''_{\text{A}}, v''_{\text{B}}) = 1·026 (\bar{v}_{\text{A}}, \bar{v}_{\text{B}}),$$

where $\bar{v}_A$ and $\bar{v}_B$ are the mean temperatures over the whole of each platinum wire. Hence we have as the connection between the temperature of the rod and the observed temperature of the platinum wires,

$$(v_A,\ v_B) = (1\cdot026+29h)(\bar{v}_A,\ \bar{v}_B) \text{ mean value.} \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

The amount of heat conducted from the sleeve* along the wire at its point of contact with the sleeve is equal to the value of $qk\,(dv/dx)$ at $x = 1$, which is

$$qkv''_0\,\alpha\frac{\cosh \alpha l}{\sinh \alpha l} = \frac{qkv''_0}{l}\cosh \alpha l \Big/ \frac{\sinh \alpha l}{\alpha l} = \frac{qkv''_0}{l}\,\frac{1\cdot161}{1\cdot053}$$

$$= \frac{0\cdot00012\times0\cdot166}{0\cdot8}\,1\cdot12v''_0 = 0\cdot000028v''_0.$$

For the two platinum wires of the upper sleeve this amounts to $0\cdot000056v_0''$, which must be deducted from the heat supplied to give the heat which flows down the rod to the platinum thermometers.

Corrections for Leads in Power Circuit.

As in the case of the platinum resistance coils, so in that of the heating coil, the No. 40 platinoid wire constituting it was not wholly wound on the sleeve, but at each end 0·8 centim. projected and served to make contact with the No. 22½ copper wires supplying current to the coil. Part, therefore, of the difference of potential measured was expended on the leads and on that part of the platinoid wire not on the sleeve. On the other hand, the passage of the current through the latter portion of the wire raised its temperature above that of the wire on the sleeve, and some of the heat generated in it would in consequence reach the sleeve by conduction along the wire. We require to determine this amount.

If W watts are spent in the platinoid wire, and L is the total length of the wire, $\frac{W}{4\cdot19L}$ gram-degrees of heat are generated per second in each centimetre of the wire. The equation for the distribution of temperature excess v along the wire in the steady state will therefore be

$$qk\frac{d^2v}{dx^2}+\frac{W}{4\cdot19L}-phv = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

where q, k, p, h have the usual meanings.

The solution of this equation may be put in the form

$$\left(\frac{W}{4\cdot19Lph}-v\right)\sinh\left(\frac{ph}{qk}\right)^{1/2} l$$

$$= \left(\frac{W}{4\cdot19Lph}-V_L\right)\sinh\left(\frac{ph}{qk}\right)^{1/2}(l-x)+\left(\frac{W}{4\cdot19Lph}-v''_{C'}\right)\sinh\left(\frac{ph}{qk}\right)^{1/2}x, \quad . \qquad (14)$$

---

* The amount conducted from the sleeve through the wooden support of the leads was too small to be taken into account.

where V_L is the temperature at the junction of the platinoid wire with the copper lead, $v''_{C'}$ that of the wire on the sleeve, x is measured from the copper lead, and l is the length of the platinoid wire between the copper lead and the sleeve.

If we neglect the heat generated by the current in the copper lead, and suppose the length of the lead to be sufficient for its temperature excess to fall to zero before the wire emerges from the containing tube, its temperature excess v_1 at a point $-x$ from the junction with the platinoid is given by the equation

$$v_1 = V_L \exp. \left(\frac{p_1 h}{q_1 k_1}\right)^{1/2} x,$$

where p_1, h_1, q_1, k_1 refer to the copper wire and have the usual meanings.

Since the flow of heat at the copper-platinoid junction is continuous, we must have, at $x = 0$,

$$qk\frac{dv}{dx} = q_1 k_1 \frac{dv_1}{dx},$$

i.e.,

$$\left\{(phqk)^{1/2} \Big/ \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}\left\{\left(\frac{W}{4{\cdot}19Lph} - V_L\right)\cosh\left(\frac{ph}{qk}\right)^{1/2} l - \left(\frac{W}{4{\cdot}19Lph} - v''_{C'}\right)\right\} = (p_1 h q_1 k_1)^{1/2} V_L.$$

Hence

$$V_L = \left\{\frac{W}{4{\cdot}19Lph}\left[\cosh\left(\frac{ph}{qk}\right)^{1/2} l - 1\right] + v''_{C'}\right\} \Big/ \left\{\cosh\left(\frac{ph}{qk}\right)^{1/2} l + \left(\frac{p_1 q_1 k_1}{pqk}\right)^{1/2} \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}.$$

The flow of heat out of the exposed platinoid wire into the sleeve is given by the value of

$$-qk\frac{dv}{dx} \text{ at } x = l,$$

i.e., by

$$\left\{(phqk)^{1/2} \Big/ \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}\left\{-\left(\frac{W}{4{\cdot}19Lph} - V_L\right) + \left(\frac{W}{4{\cdot}19Lph} - v''_{C'}\right)\cosh\left(\frac{ph}{qk}\right)^{1/2} l\right\}$$

$$= \left\{(phqk)^{1/2} \Big/ \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}$$

$$\left\{-\left[\frac{W}{4{\cdot}19Lph} - \left\{\frac{W}{4{\cdot}19Lph}\left(\cosh\left(\frac{ph}{qk}\right)^{1/2} l - 1\right) + v''_{C'}\right\} \Big/ \left\{\cosh\left(\frac{ph}{qk}\right)^{1/2} l + \left(\frac{p_1 q_1 k_1}{pqk}\right)^{1/2} \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}\right]\right.$$
$$\left. + \left(\frac{W}{4{\cdot}19Lph} - v''_{C'}\right)\cosh\left(\frac{ph}{qk}\right)^{1/2} l\right\}.$$

The total flow into the sleeve from the two wires will therefore be

$$= 2\left\{(phqk)^{1/2} \Big/ \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}$$

$$\left\{-\left\{\frac{W}{4{\cdot}19Lph}\left[\left(\frac{p_1 q_1 k_1}{pqk}\right)^{1/2} \sinh\left(\frac{ph}{qk}\right)^{1/2} l + 1\right] + v''_{C'}\right\} \Big/ \left\{\cosh\left(\frac{ph}{qk}\right)^{1/2} l + \left(\frac{p_1 q_1 k_1}{pqk}\right)^{1/2} \sinh\left(\frac{ph}{qk}\right)^{1/2} l\right\}\right.$$
$$\left. + \left(\frac{W}{4{\cdot}19Lph} - v''_{C'}\right)\cosh\left(\frac{ph}{qk}\right)^{1/2} l\right\} \quad . \quad (15)$$

For the platinoid wire the values of the constants are $q = 0{\cdot}00012$ sq. centim., $k = 0{\cdot}050$ (mean), $p = 0{\cdot}046$ centim. over silk, $h = 0{\cdot}00022$ (mean).

Hence

$$\left(\frac{ph}{qk}\right)^{1/2} = (1{\cdot}67)^{1/2} = 1{\cdot}29,\ \left(\frac{ph}{qk}\right)^{1/2} l = 1{\cdot}03,\ (pqk)^{1/2} = 5{\cdot}2\times10^{-4},\ \text{and}\ (phqk)^{1/2} = 78\times10^{-7}.$$

$$4{\cdot}19\mathrm{L}ph = 1{\cdot}31\times10^{-3}.$$

For the copper wire, $q_1 = 0{\cdot}0033$ sq. centim., $k_1 = 1{\cdot}00$ (mean), $p_1 = 0{\cdot}27$ centim. over silk.

Hence

$$(p_1q_1k_1)^{1/2} = (9\times10^{-4})^{1/2} = 0{\cdot}030.$$

The flow of heat into the sleeve from the two wires per second will therefore be

$$= 0{\cdot}0060\mathrm{W} - 0{\cdot}000020v''_{\mathrm{C}'}.$$

Thus of W watts spent in the wire, only

$$\left(\frac{29{\cdot}0}{30{\cdot}6} + 0{\cdot}0060\right)\mathrm{W} - 0{\cdot}000020v''_{\mathrm{C}'},$$

i.e.,

$$0{\cdot}954\mathrm{W} - 0{\cdot}000020v''_{\mathrm{C}'}$$

reach the sleeve, where $v''_{\mathrm{C}'}$ is the excess of the mean temperature of the sleeve over that of the surrounding tube.

The watts were measured at the ends of the copper leads, for the resistance of which a further correction must be made, a correction which will depend on the temperature of the leads and therefore of the tube. Each lead of copper wire, 0·065 centim. diameter, was 55 centims. long, and of this length 33 centims. was in the Dewar tube, 22 centims. in air. The resistance per metre at the temperature of the air, 15° to 18° C., may be taken as 0·051 ohm, and the total resistance of the leads as 0·056 ohm at the temperature of the air. When the platinum temperature of the Dewar tube is τ, we may, with sufficient accuracy, take the resistance of the leads to be

$$0{\cdot}056\left(\frac{2}{5} + \frac{3}{5}\frac{\tau}{290}\right)\text{ ohm}$$

$$= 0{\cdot}022 + 0{\cdot}000116\tau.$$

If, therefore, A amperes flow through the coil, and give a difference of potential of P volts at the terminals, the watts W spent in the platinoid wire

$$= \mathrm{A}\,\{\mathrm{P} - (0{\cdot}022 + 0{\cdot}000116\tau)\,\mathrm{A}\}.$$

Hence the watts expended on the sleeve

$$= 0{\cdot}954\mathrm{A}\,\{\mathrm{P} - (0{\cdot}022 + 0{\cdot}000116\tau)\,\mathrm{A}\} - 0{\cdot}000020v''_{\mathrm{C}'}, \quad . \quad . \quad . \quad (16)$$

where $v''_{\mathrm{C}'}$ is the excess of the mean temperature of the sleeve over that, τ, of the tube.

It has been stated that the power supplied was measured by an ammeter in circuit and by applying a voltmeter to the ends of the heating coil for the few seconds necessary to take a reading. Owing to the resistance of the voltameter, $R_1 = 113$ ohms at 14° C., not being very great compared to that of the heating coil, $R_0 = 6{\cdot}4$ ohms at 18° C., the measured value P_0 of the volts at the ends of the heating coil is less than the value P when the voltameter is not in circuit, and the readings obtained must, if r is the resistance external to the coil, and the electromotive force in circuit is constant, be multiplied by

$$\frac{R_0+R_1}{R_1}\,\frac{r+R_0R_1/(R_0+R_1)}{r+R_0}$$

to give the true volts on the coil, *i.e.*,

$$\text{the volts P on the coil} = P_0\left(1+\frac{R_0}{R_1}\right)\left(1-\frac{R_0^{\,2}}{(R_0+R_1)\,(r+R_0)}\right).$$

Since the current was derived throughout from three storage cells kept well charged, it will be sufficient in the factor involving r to write $r+R_0 = 6/A$, and we have

$$\text{volts P on coil} = P_0\left(1+\frac{6{\cdot}4}{113}\right)\left(1-\frac{(6{\cdot}4)^2}{119{\cdot}4\times 6}A\right) = 1{\cdot}057P_0\,(1-0{\cdot}057A).$$

Hence the watts spent on the sleeve

$$= 0{\cdot}954A\,\{1{\cdot}057\,(1-0{\cdot}057A)\,P_0-(0{\cdot}022+0{\cdot}000116\tau)\,A\}-0{\cdot}000020v''_{C'}$$
$$= 1{\cdot}008A\,\{(1-0{\cdot}057A)\,P_0-(0{\cdot}021+0{\cdot}00011\tau)\,A\}-0{\cdot}000020v''_{C'}.$$

Hence H, the total quantity of heat imparted to the bar per second, is given by the equation*

$$4{\cdot}19H = 1{\cdot}008A\,\{(1-0{\cdot}057A)\,P_0-(0{\cdot}021+0{\cdot}00011\tau)\,A\}-0{\cdot}000020v''_{C'}-0{\cdot}000056v''_{B}.$$

Or, since $(v''_B,\ v''_C) = (v_B,\ v_C)$ very nearly,

$$4{\cdot}19H = 1{\cdot}008A\,\{(1-0{\cdot}057A)\,P_0-(0{\cdot}021+0{\cdot}00011\tau)\,A\}-0{\cdot}000020v_{C'}-0{\cdot}000056v_{B}.$$

Hence

$$H = 0{\cdot}2406A\,\{(1-0{\cdot}057A)\,P_0-(0{\cdot}021+0{\cdot}00011\tau)\,A\}-0{\cdot}0000048v_{C'}-0{\cdot}000013v_{B}. \quad (17)$$

In the small term $0{\cdot}000013v_B$ it will be sufficient if we write $v_B = v_{C'}\,x_B/x_{C'}$.

Thus, for the original equation $Q = H-shv_{C'}$, p. 386, we must substitute

$$Q = 0{\cdot}2406A\,\{(1-0{\cdot}057A)\,P_0-(0{\cdot}021+0{\cdot}00011\tau)\,A\}$$
$$-\left(sh+0{\cdot}0000048+0{\cdot}000013\,\frac{x_B}{x_{C'}}\right)v_{C'}.$$

---

* For simplicity, the loss of heat along the wires of the upper temperature measuring sleeve, p. 392, is treated as if it took place from the heating sleeve.

Hence in the equation (4) for k (p. 386) we must write for H

$$0{\cdot}2406\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\},$$

and for sh

$$sh+0{\cdot}0000048+0{\cdot}000013\,\frac{x_{\mathrm{B}}}{x_{\mathrm{C}'}}.$$

We thus obtain

$$kq\cosh\left(\frac{ph}{qk}\right)^{1/2}x_{\mathrm{C}}=\frac{0{\cdot}2406\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\}}{v_{\mathrm{B}}-v_{\mathrm{A}}}(x_{\mathrm{B}}\mathrm{S}_{\mathrm{B}}-x_{\mathrm{A}}\mathrm{S}_{\mathrm{A}})$$
$$-\left(sh+0{\cdot}0000048+0{\cdot}000013\,\frac{x_{\mathrm{B}}}{x_{\mathrm{C}'}}\right)x_{\mathrm{C}'}\mathrm{S}_{\mathrm{C}'}.$$

Finally, substituting for the temperatures of the rod v_{B} and v_{A} in terms of $\bar{v}_{\mathrm{B}}$ and $\bar{v}_{\mathrm{A}}$ the observed mean temperatures of the platinum resistances by means of equation (12), p. 392, we have

$$kq\cosh\left(\frac{ph}{qk}\right)^{1/2}x_{\mathrm{C}}=\frac{0{\cdot}2406\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\}}{(1{\cdot}026+29h)\,(\bar{v}_{\mathrm{B}}-\bar{v}_{\mathrm{A}})}(x_{\mathrm{B}}\mathrm{S}_{\mathrm{B}}-x_{\mathrm{A}}\mathrm{S}_{\mathrm{A}})$$
$$-\left(sh+0{\cdot}0000048+0{\cdot}000013\,\frac{x_{\mathrm{B}}}{x_{\mathrm{C}'}}\right)x_{\mathrm{C}'}\mathrm{S}_{\mathrm{C}'},$$

that is,

$$kq\cosh\left(\frac{ph}{qk}\right)^{1/2}x_{\mathrm{C}}$$
$$=\frac{(0{\cdot}2345-28h)\,\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\}}{\bar{v}_{\mathrm{B}}-\bar{v}_{\mathrm{A}}}(x_{\mathrm{B}}\mathrm{S}_{\mathrm{B}}-x_{\mathrm{A}}\mathrm{S}_{\mathrm{A}})$$
$$-\left(sh+0{\cdot}0000048+0{\cdot}000013\,\frac{x_{\mathrm{B}}}{x_{\mathrm{C}'}}\right)x_{\mathrm{C}'}\mathrm{S}_{\mathrm{C}'},\qquad(18)$$

where $s = 1{\cdot}47+0{\cdot}36+0{\cdot}11 = 1{\cdot}94$ sq. centims.

Effect of the use of the Variable State of Temperature Distribution.

So far the theory has been worked out on the assumption that the temperature distribution throughout the apparatus is steady. But it has been stated that the measurements were made during the gradual rise of temperature from that of liquid air. We have therefore to consider the effect of this on our fundamental equation (4).

Let v be the temperature of the bar at a cross section distant x from that where the bar joins the enclosing tube, and let V be the temperature of the tube. Then v satisfies the conditions

$$\left.\begin{array}{c} qcp\,\dfrac{\partial v}{\partial t}=qk\,\dfrac{\partial^2 v}{\partial x^2}-ph\,(v-\mathrm{V}),\\ \mathrm{V}=f(t),\quad v=\mathrm{V}\text{ at }x=0,\\ qk\,\dfrac{\partial v}{\partial x}+ph\,(v-\mathrm{V})=\mathrm{Q}\text{ at }x=x_{\mathrm{C}},\text{ the free end of the bar,}\end{array}\right\}$$

the heat Q being supposed imparted to the bar at its free end.

Let $v = v_0 + v_1$, where v_0 is independent of t, and $v_0 + v_1$ satisfy the following conditions,

$$\left.\begin{aligned} 0 &= qk\frac{\partial^2 v_0}{\partial x^2} - phv_0, \\ v_0 &= 0 \text{ at } x = 0, \\ qk\frac{\partial v_0}{\partial x} + phv_0 &= \mathrm{Q} \text{ at } x = x_{\mathrm{C}}, \end{aligned}\right\}$$

$$\left.\begin{aligned} qc\rho\frac{\partial v_1}{\partial t} &= qk\frac{\partial^2 v_1}{\partial x^2} - ph\,(v_1 - \mathrm{V}), \\ \mathrm{V} = f(t), \quad v_1 &= \mathrm{V} \text{ at } x = 0, \\ qk\frac{\partial v_1}{\partial x} + ph\,(v_1 - \mathrm{V}) &= 0 \text{ at } x = x_{\mathrm{C}}, \end{aligned}\right\}$$

then $v_0 + v_1$ obviously satisfies the conditions laid down for v. The solution of the equations for v_0 is

$$v_0 = \frac{\mathrm{Q}}{(phqk)^{1/2}} \sinh\left(\frac{ph}{qk}\right)^{1/2} x \Big/ \left[\cosh\left(\frac{ph}{qk}\right)^{1/2} x_{\mathrm{C}} + \left(\frac{ph}{qk}\right)^{1/2} \sinh\left(\frac{ph}{qk}\right)^{1/2} x_{\mathrm{C}}\right].$$

Hence $v_0 = 0$ when $\mathrm{Q} = 0$.

Since v_1 is independent of Q, we have at any instant for all values of x

$$v_1 \text{ with Q finite} - v_1 \text{ with Q zero} = v_0.$$

Since, however, v_1 cannot be observed at the same instant with Q zero and Q finite, temperatures are observed alternately with Q zero and Q finite, and the distribution with Q finite compared with the distribution with Q zero calculated for the same instant by interpolation between the observations with Q zero made immediately before and after.

In all but three or four cases it was only necessary to take the mean of the observations with Q zero taken before and after.

Since $f(t)$ is a continuous function the error committed by this method of observation will be very small.

Standardisation of Platinum Thermometers.

As the observations of conductivity were made over the range of temperature between about $-180°$ C. and $30°$ C., the temperatures selected for standardising the thermometers used were the boiling point of liquid oxygen, the freezing point and the boiling point of water.

The two thermometers were slipped on to a short rod of brass 2 centims. in length and of the same diameter as the rods used throughout the work. The rod and thermometers were then placed in a short brass tube about 3 centims. long, one end of which was closed by a brass plate. Through a cork in the other end of it the thermometer leads and a pair of compensating leads similar to those of the thermometers passed to a series of mercury cups, which allowed either of the thermometers or the compensating leads to be connected to a resistance box.

The coils of this box were of platinum silver wire, and were multiples of the B.A. ohm at 15°·5 C. The fixed arms of the resistance bridge were the 10- and 1000-ohm coils of the box respectively, and any error in the ratio of these two was eliminated by substituting a 1- or 2-ohm standard resistance coil for the thermometers and finding its apparent resistance. This enables the box readings to be converted to true ohms.

The following is a record of one of the tests.

Hartmann and Braun 2-ohm standard correct at 17°·3 C. Temperature coefficient 0·0002. Temperature of room, 15°·5 C. Hence standard = 1·9993 ohm.

Standard + leads = 2·0477 apparent ohms of box.
Leads = 0·0266 ,, ohm ,,
Standard 1·9993 ohms = 2·0211 ,, ohms ,,

Other tests agreed to within one part in 5000.

The resistances recorded in what follows have all been reduced to standard ohms.

The current used in the bridge was derived from a pint Leclanché cell connected to the bridge through a resistance of 53 ohms.

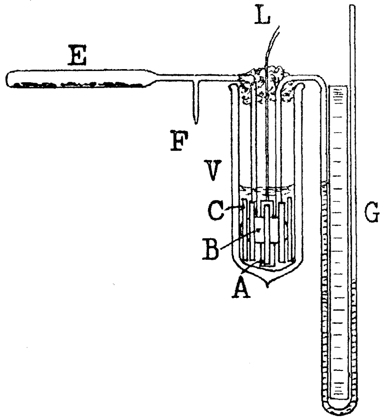

Fig. 2.

The test of the resistances of the measuring coils at the low temperature was made with the arrangement shown in fig. 2.

A is the short brass cylinder containing the measuring coils, the leads L of which, and the compensating leads, pass through a cork closing the top of the cylinder. B is the bulb of the oxygen vapour-pressure thermometer* EFG used to determine the temperature on the constant-volume hydrogen scale of the liquid air in the Dewar vessel V.

C represents six short copper rods placed round A and B to keep the temperature of the liquid air uniform. E is a bulb containing crystals of potassium permanganate which on heating furnish the oxygen. F is the sealed tube through which the thermometer was exhausted before use. G is the mercury gauge to determine the vapour pressure of the oxygen condensed in the bulb B.

The difference of level of the mercury in G was initially 76·73 centims., and the height of the barometer 76·77 centims.

Liquid air having been poured into the Dewar tube V till it was almost full, and the opening of the tube plugged with cotton wool, the bulb E was heated so as to generate the oxygen, which condensed in B. The vapour pressure indicated initially

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* I adopted this method of standardisation on the suggestion of Dr. M. W. Travers, F.R.S., and found it most convenient.

was 31·32 centims., corresponding, according to TRAVERS' determinations, to 82°·4 C. absolute, and rose during the first 40 minutes to 33·61 centims., corresponding to a temperature of 82°·9 C. During the next 16 minutes the following measurements were taken :—

Elliott B.A. box, ratio arms 10 : 1000.

h.	m.		
11	52	manometer	43·16 centims.
11	54	compensating leads	0·0964 ohm (apparent).
11	56	lower coil	0·8285 ,, ,,
11	58	upper ,,	0·8332 ,, ,,
12	0	manometer	42·77 centims.
12	2	upper coil	0·8335 ohm (apparent).
12	4	lower ,,	0·8295 ,, ,,
12	6	compensating leads	0·0967 ,, ,,
12	8	manometer	42·51 centims.

Barometer 76·79 centims., temperature 15°·5 C.

From these observations we deduce that, at a mean pressure of 33·99 centims. of mercury at 15°·5 C., or 33·92 centims. of mercury at 0° C., at sea-level in latitude 45°, under which oxygen boils at 82°·9 C. absolute on the constant volume hydrogen thermometer, the resistances of the coils were :—

Upper coil . . 0·8333−0·0966 = 0·7367 apparent ohm = 0·7286 true ohm.
Lower ,, . . 0·8290−0·0966 = 0·7324 ,, ,, = 0·7243 ,, ,,

The determinations of resistance at the freezing point of water were made by placing the brass tube containing the coils at a depth of about 5 centims. in clean ice shavings contained in a brass can with holes in the bottom to allow the ice to drain. After the apparatus had stood for half an hour the following observations were made :—

h.	m.		
1	0	compensating leads	0·1152 ohm (apparent).
1	2	lower coil	2·877 ohms ,,
1	4	upper ,,	2·882 ,, ,,
1	6	,, ,,	2·882 ,, ,,
1	8	lower ,,	2·877 ,, ,,
1	10	compensating leads	0·1153 ohm ,,

From these observations we deduce that the resistances of the coils at the melting point of ice were :—

Upper coil . . 2·882−0·115 = 2·767 apparent ohms = 2·736 true ohms.
Lower ,, . . 2·877−0·115 = 2·762 ,, ,, = 2·731 ,, ,,

In the resistance determinations at the boiling point of water the coils with the brass rod through them were placed in a narrow brass tube 12 centims. long, the top closed by a cork through which the leads passed, and the whole placed in a hypsometer with the lower 11 centims. of the tube exposed to the steam. Owing to the uncertain behaviour of the shellac on which the insulation of the coils depended in the presence of moisture at this temperature, the coils were well dried by being kept at 100° C. for about an hour and a half before the following measurements were taken:—

h.	m.			
3	40	compensating leads . .	0·1223 ohm	(apparent).
3	42	lower coil	3·874 ohms	,,
3	44	upper ,, ...	3·878 ,,	,,
3	46	,, ,, ...	3·878 ,,	,,
3	48	lower ,, ...	3·876 ,,	,,
3	50	compensating leads . .	0·1219 ohm	,,

Barometer 76·78 centims. at 15°·5 C. = 76·63 centims. at 0° C. at sea-level in latitude 45°.

From these measurements we conclude that the resistances of the coils at a temperature of 100°·23 C. are:—

Upper coil . . 3·878 − 0·122 = 3·756 apparent ohms = 3·715 true ohms.
Lower ,, . . 3·875 − 0·122 = 3·753 ,, ,, = 3·712 ,, ,,

Although the slight excess of the resistance of the upper coil over that of the lower coil is not proportional to that of either coil, it is so small in amount that it will be sufficient in defining the platinum temperature scale for the coils used and in reducing it to the standard hydrogen scale to use the means of the observed resistances at each temperature.

We thus have for the platinum temperature t_P corresponding to a resistance R of either coil

$$t_P = \frac{R - 2{\cdot}734}{0{\cdot}980}\, 100^\circ{\cdot}23 = 102{\cdot}3\,(R - 2{\cdot}734), \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

and as the basis of the connection between the platinum and hydrogen scales the following:—

Temperature t_H on hydrogen scale.			Resistance.
82°·9 C. absolute	=	−190°·1 C.	0·726 ohm.
273°·0 C.* ,,	=	0° C.	2·734 ohms.
373°·23 C. ,,	=	100°·23 C.	3·714 ,,

In order to compare the platinum used with that used by other experimenters, the "fundamental constants" of CALLENDAR are calculated as follows:—

$$\text{“Fundamental coefficient”} = \frac{3{\cdot}714 - 2{\cdot}734}{2{\cdot}734 \times 100{\cdot}23} = \frac{0{\cdot}980}{274{\cdot}0} = 0{\cdot}00358.$$

$$\text{“Fundamental zero”} = -\frac{1}{0{\cdot}00358} = -279^\circ{\cdot}3.$$

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* CHAPPUIS, 'Travaux et Mémoires du Bureau International,' VI., p. 108.

The temperature on the platinum thermometer corresponding to $-190^\circ\cdot1$ C. or $82^\circ\cdot9$ C. absolute on the hydrogen thermometer is

$$\frac{0\cdot726-2\cdot734}{2\cdot734\times0\cdot00358}=-205^\circ\cdot2.$$

The δ of the difference equation

$$t_H - t_P = \delta\left(\frac{t_H}{100}-1\right)\frac{t_H}{100} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (20)$$

is then given by

$$15\cdot1 = \delta\left(-\frac{190}{100}-1\right)\left(-\frac{190}{100}\right), \quad \text{or} \quad \delta = \frac{15\cdot1}{5\cdot51} = 2\cdot74.$$

The platinum wire is, therefore, nearly identical in properties with that used as a standard by DEWAR and FLEMING,[*] the fundamental constants of which CALLENDAR has calculated,[†] and appears to be rather less pure than that used by me in the determination of the heat conductivities of some substances which are electrical insulators at low temperatures.[‡]

The connection between the two temperature scales (20), and between the conductivities according to the two scales,

$$k_H = k_P\frac{dt_P}{dt_H}, \ i.e., \ k_H = k_P\{1+\delta(0\cdot01-t_H/50{,}000)\}, \quad . \ . \ . \ . \quad (21)$$

is shown in the following table[§]:—

TABLE of Connection between Temperatures and Thermal Conductivities on the Platinum Scale used and the Hydrogen Scale.

t_H.	t_P.	k_H/k_P.	t_H.	t_P.	k_H/k_P.
°	°		°	°	
100	100	0·973	− 60	− 62·6	1·060
90	90·2	0·978	− 70	− 73·3	1·066
80	80·4	0·984	− 80	− 84·0	1·071
70	70·6	0·989	− 90	− 94·7	1·077
60	60·7	0·995	− 100	− 105·5	1·082
50	50·7	1·000	− 110	− 116·3	1·088
40	40·7	1·006	− 120	− 127·2	1·093
30	30·6	1·011	− 130	− 138·2	1·099
20	20·4	1·016	− 140	− 149·2	1·104
10	10·2	1·022	− 150	− 160·3	1·110
0	0	1·027	− 160	− 171·4	1·115
− 10	− 10·3	1·033	− 170	− 182·6	1·121
− 20	− 20·7	1·038	− 180	− 193·8	1·126
− 30	− 31·1	1·044	− 190	− 205·1	1·132
− 40	− 41·5	1·049	− 200	− 216·4	1·137
− 50	− 52·1	1·056			

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[*] 'Phil. Mag.,' 40, p. 100 (1895).

[†] 'Phil. Mag.,' 47, p. 213 (1899).

[‡] 'Phil. Trans.,' A, 204, p. 451 (1905).

[§] Compare CALLENDAR, 'Phil. Mag.,' 47, p. 214 (1899).

Standardisation of Resistance Bridge.

The standardisation of the resistance bridge used in determining the resistances of the temperature coils during a conductivity experiment was effected by substituting standard coils of 0·1, 0·2, 0·5, 1·0, and 2·0 ohms for one of the temperature coils, and adjusting the bridge to a balance as in the experiments.

The standardisation of each of the resistances of the dial used in the measurement of the difference of the resistances of the two platinum coils, from which the difference of temperature of the two points x_A and x_B of the test bar is calculated, was effected by substituting for it a standard coil of 0·01006 ohm by Hartmann and Braun, and examining the effect of the substitution on the balance of the bridge.

The resistance, 0·1055 ohm, found by adding together the resistances of the ten coils found in this way, agrees very well with the value 0·1057 found by direct comparison of the dial with a Wolff 0·1-ohm standard after the experiments were completed.

In the tables of observations which follow, the values of the resistances given are the corrected ones.

Standardisation of Ammeter and Voltmeter.

The Weston milliamperemeter No. 5491, which was used throughout the work to measure the current through the coil in which the heat was generated, was standardised by comparison with a Kelvin balance which had been checked by the copper voltmeter. It was found to read 1 per cent. too high. The values of the current given in the following tables are the corrected values.

The Keiser and Schmidt moving-coil voltmeter was compared by a potentiometer method with a cadmium and a Clark cell, either separately, in series, or in opposition. The electromotive force of each cell had been determined by comparison with a standard Clark. It was found to read 0·8 per cent. too high. The values given in the following tables are corrected values.

Determination of the Coefficient "h" of Loss of Heat from the Surface of the Rod.

The final expression for the thermal conductivity k of the material of the rods, equation (18), p. 396, involves a knowledge of h, the heat lost in 1 second from 1 sq. centim. of surface of the bar when that area is 1° C. hotter than the surrounding tube. As the terms involving h only affect the result to a small extent, an approximate value of the quantity is sufficient.

To determine this value, the heating coil and upper temperature measuring coil were placed on a brass rod 5·2 centims. long, so as to divide the rod into three equal parts. This rod with its coils was suspended in the centre of the tube. A short brass rod 2 centims. long was fitted into the base of the tube, and on the end projecting into the tube the lower temperature measuring coil was placed. The coils were connected up as in an experiment to determine conductivity.

On sending a small current through the heating coil the temperature of the rod in the centre is raised, and when equilibrium between the electrical supply of heat and the loss of heat from the surface of the rod had been attained, the excess of temperature of the rod over the temperature of the shorter one continuous with the enclosing tube was observed. The temperature of the latter and the supply of power to the central rod were also measured. To get rid of the effect of the slow change of temperature of the enclosure the heating current was then switched from the rod to the enclosure, and the difference of temperature between rod and enclosure again measured after the steady state had been attained.

The same series of observations was made at about 100°, 200° and 300° absolute temperature.

The total quantity of heat, H, imparted to the bar per second by the heating coil is given in gram-degrees by equation (17), p. 395, as

$$\mathrm{H} = 0{\cdot}2406\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\,\mathrm{A}\}-0{\cdot}0000048v_{\mathrm{C}'}-0{\cdot}000013v_{\mathrm{B}},$$

where A = amperes sent through coil,

P_0 = volts at ends of leads,

τ = absolute temperature of enclosure.

$v_{\mathrm{C}'} = v_{\mathrm{B}}$ = excess of temperature of rod over that of enclosure.

If S is the total surface of rod and sleeves, and the coefficient h of loss from surfaces of rod and sleeve be assumed to be the same, since both were slightly oiled surfaces, we have

$$\mathrm{H} = \mathrm{S}hv_{\mathrm{B}}.$$

Hence

$$h\mathrm{S} = 0{\cdot}2406\mathrm{A}\,\{(1-0{\cdot}057\mathrm{A})\,\mathrm{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\,\mathrm{A}\}\,v_{\mathrm{B}}^{-1}-0{\cdot}000018.$$

Now

$$\begin{aligned} \mathrm{S} &= 2\,\{(2{\cdot}28\times0{\cdot}42)+(2{\cdot}14\times0{\cdot}31)\} \text{ for sides and ends of sleeves,}\\ &\quad +(5{\cdot}2-1{\cdot}3)\,1{\cdot}84+(2\times0{\cdot}27) \quad ,, \quad ,, \quad ,, \quad \text{rod,}\\ &= 10{\cdot}96 \text{ sq. centims.} \end{aligned}$$

$$v_{\mathrm{B}} = (1{\cdot}026+28{\cdot}6h)\,\bar{v}_{\mathrm{B}} \quad (12),\ \text{p. } 392, \qquad \text{and} \qquad \bar{v}_{\mathrm{B}} = 102{\cdot}3\,(\mathrm{R}_{\mathrm{B}}-\mathrm{R}_\tau) \quad (19),\ \text{p. } 400,$$

where $\bar{v}_{\mathrm{B}}$ is the mean temperature of the platinum temperature coil on the rod, R_{B} its resistance, R_τ that of the coil in thermal contact with the enclosure.

Thus

$$h = \frac{1}{10{\cdot}96}\left\{\frac{0{\cdot}2406\text{A}\left[(1-0{\cdot}057\text{A})\,\text{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\,\text{A}\right]}{(1{\cdot}026+29h)\;102{\cdot}35\,(\text{R}_\text{B}-\text{R}_\tau)}-0{\cdot}000018\right\},$$

i.e.,

$$h = 0{\cdot}000214\text{A}\,\frac{(1-0{\cdot}057\text{A})\,\text{P}_0-(0{\cdot}021+0{\cdot}00011\tau)\,\text{A}}{(1{\cdot}026+29h)\,(\text{R}_\text{B}-\text{R}_\tau)}-0{\cdot}0000016. \qquad (22)$$

The table on p. 405 gives the observations and their reduction.

Although the determination of the general law of variation of the coefficient h with temperature is not necessary for the purpose of the present work, it may be of interest to point out that the observed values of h for the three values of the mean temperature of rod and enclosure measured on the hydrogen temperature scale do not agree with any of the formulæ which have been proposed to express that variation. LORENZ[*] proposed the formula $h \propto 1+bt^{1/4}$, but WAGNER[†] has recently found it unsuitable and suggests $h \propto 1+bt^2$, while I found[‡] $h \propto t^n$, where $n = 0{\cdot}2\dot{6}$, suitable over a comparatively small range of temperature. The present observations suggest the expression $h \propto 1+bt^{3/2}$ for the case of a small rod at very low temperatures in an enclosure only a few, say 10, degrees cooler than itself. t is the mean temperature of rod and enclosure on the hydrogen thermometer scale.

Results of Thermal Conductivity Experiments.

In the Tables of Observations and Results which follow, the values of the quantities observed are given after correction for errors, if any, of the observing instruments. In addition, a few of the most important quantities involved in the calculation of the conductivities are given, in order to facilitate comparison of the different experiments with each other, and to enable errors in the calculations to be more readily detected.

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[*] L. LORENZ, 'Ann. der Physik,' 13, p. 582 (1881).

[†] R. WAGNER, 'Beiblätter,' 27, p. 534 (1903).

[‡] C. H. LEES, 'Phil. Mag.,' 28, p. 429 (1889).

TABLE of Observations for h.

τ, abs. Pt. scale.	A.	P_0.	$(1-0\cdot057\,A)\times P_0$.	$(0\cdot021+0\cdot00011\tau)\,A$.	$(1-0\cdot057A)\,P_0-(0\cdot021+0\cdot00011\tau)\,A$.	$A\,\{(1-0\cdot057A)\,P_0-(0\cdot021+0\cdot00011\tau)\,A\}$.	Difference of resistance of two coils observed.	R_B-R_τ.	$1\cdot026+29h$.	$(R_B-R_\tau)\times(1\cdot026+29h)$.	$A\,\dfrac{(1-0\cdot057A)\,P_0-(0\cdot021+0\cdot00011\tau)\,A}{(1\cdot026+29h)\,(R_B-R_\tau)}$.	$0\cdot000214A\,\dfrac{(1-0\cdot057A)\,P_0-(0\cdot021+0\cdot00011\tau)\,A}{(1\cdot026+29h)\,(R_B-R_\tau)}$.	h.	Mean absolute platinum temperature of rod and enclosure.	Mean temperature of rod and enclosure on hydrogen scale.
° C.	amp.	volt					ohm	ohm						° C.	° C.
298	·120	·720	·715	·006	·709	·0851	·0915	·0873	1·032	·0901	·945	·000202	·000200	302·5	+ 23
298	0						·0042								
228	0						− ·0437								
206	·123	·723	·718	·005	·713	·0877	·0698	·1123	1·031	·1158	·758	·000162	·000160	212·0	− 64
206	0						− ·0425 calculated								
106	·122	·696	·691	·004	·687	·0838	·0987	·1357	1·030	·1398	·599	·000128	·000126	113·0	− 155
106	0						− ·0370 calculated								
96	0						− ·0365								

COPPER ROD.

Turned from a Length of Soft-drawn High-conductivity Copper Conductor. Density* at 23° C. = 8·84.

$a_2 = 3{\cdot}65$ centims., approx. $k = {\cdot}9$, therefore $b' = {\cdot}32$, $x_A = {\cdot}42$, $x_B = 4{\cdot}71$, $x_C = 5{\cdot}13$, $x_{C'} = 5{\cdot}35$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.†	II.†	III.†	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 27	·340	2·084	·688	·157	·0310	·0268	2·74	·0572	·039	4·31	·247	·245	·273	·898	+ 26	·910
+ 25	0				·0042											
− 189	0				− ·0017											
− 179	·348	2·020	·685	·158	·0229	·0246	2·52	·0627	·031	4·31	·269	·267	·271	·985	− 166	1·105
− 176	0				− ·0017											
− 166	·347	2·022	·684	·158	·0243	·0256	2·62	·0603	·031	4·31	·260	·258	·271	·952	− 154	1·061
− 157	0				− ·0010											
− 147	·347	2·030	·686	·158	·0252	·0260	2·66	·0593	·032	4·31	·256	·254	·271	·937	− 137	1·036
− 144	0				− ·0007											
− 136	·347	2·032	·686	·158	·0260	·0269	2·75	·0573	·032	4·31	·247	·245	·271	·905	− 127	·996
− 133	0				− ·0010											
− 125	·346	2·036	·685	·158	·0261	·0269	2·75	·0573	·033	4·31	·247	·245	·272	·901	− 118	·989
− 122	0				− ·0008											
− 116	·346	2·040	·687	·158	·0261	·0269	2·75	·0573	·034	4·31	·247	·245	·272	·901	− 109	·982
− 113	0				− ·0011											
− 98	·346	2·044	·688	·158	·0261	·0269	2·75	·0573	·034	4·31	·247	·245	·272	·901	− 93	·974
− 96	0				− ·0008											
− 87	·346	2·050	·690	·159	·0267	·0272	2·78	·0571	·034	4·31	·246	·244	·272	·898	− 83	·965
− 83	0				− ·0003											
− 73	·345	2·060	·691	·159	·0271	·0273	2·79	·0569	·035	4·31	·245	·243	·272	·894	− 70	·955
− 71	0				− ·0002											
− 62	·345	2·060	·691	·158	·0272	·0272	2·78	·0567	·035	4·31	·245	·243	·272	·894	− 60	·950
− 56	0				·0002											
− 47	·344	2·064	·691	·158	·0278	·0273	2·79	·0569	·036	4·31	·245	·243	·272	·894	− 45	·941
− 44	0				·0006											
− 39	0				·0002											

− 31	·346	2·100	·707	·162	·0281	·0276	2·82	·0573	·037	4·31	·247	·245	·273	·898	− 30	·939
− 24	0				·0008											
− 15	·346	2·110	·710	·163	·0288	·0278	2·85	·0571	·037	4·31	·246	·244	·273	·894	− 15	·927
− 11	0				·0012											
− 4	·346	2·110	·710	·163	·0296	·0280	2·87	·0567	·038	4·31	·245	·243	·273	·891	− 4	·918
0	0				·0021											
+ 7	·345	2·114	·709	·163	·0298	·0275	2·81	·0579	·038	4·31	·250	·248	·273	·909	+ 7	·932
9	0				·0026											
16	·345	2·120	·711	·162	·0303	·0276	2·82	·0573	·039	4·31	·247	·245	·273	·898	16	·915
19	0				·0028											
25	·345	2·120	711	·162	·0305	·0275	2·81	·0575	·039	4·31	·248	·246	·273	·902	25	·915
27	0				·0032											

Duration of experiment, 6 hours.

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* For the determinations of density throughout I have to thank one of my senior students at the East London College, Miss F. VERINDER, B.Sc.

† Column I. $= \dfrac{\text{Heat supplied to rod per second}}{\text{Temperature difference}} (x_B S_B - x_A S_A)$.

Column II. $=$ Column I. $- \left(sh + \cdot 0000048 + \cdot 00001 \dfrac{x_B}{x_{C'}}\right) x_{C'} S_{C'}$.

Column III. $= q \cosh \left(\dfrac{ph}{qk}\right)^{1/2} x_C$.

SILVER ROD.

Turned from a Rod of Fine Silver ·999, supplied by Messrs. JOHNSON & MATTHEY. Density at 21° = 10·47.

$a_2 = 4\cdot35$ centims., approx $k = 1\cdot0$, therefore $b' = \cdot32$, $x_A = \cdot42$, $x_B = 5\cdot41$, $x_C = 5\cdot83$, $x_{C'} = 6\cdot05$ centims., $s = 1\cdot83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6\cdot85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.																
+ 28	0				+ ·0034										° C.	
33	·352	2·180	·744	·169	·0348	·0315	3·22	·0526	·037	5·03	·264	·261	·275	·951	+ 33	·961
31	0				·0033											
− 192	0				·0022											
− 176	·366	2·124	·757	·175	·0387	·0353	3·61	·0485	·029	5·01	·243	·241	·272	·885	− 163	·992
− 174	0				·0046											
− 169	0				·0001											
− 159	·366	2·140	·764	·175	·0351	·0347	3·55	·0493	·030	5·01	·248	·246	·272	·904	− 148	1·005
− 154	0				·0009											
− 142	·366	2·150	·767	·176	·0356	·0346	3·54	·0497	·030	5·01	·250	·248	·272	·910	− 133	1·006
− 139	0				·0012											
− 132	·365	2·141	·765	·176	·0358	·0345	3·53	·0499	·031	5·02	·251	·249	·272	·914	− 124	1·004
− 130	0				·0015											
− 120	·365	2·160	·769	·177	·0359	·0343	3·51	·0505	·031	5·02	·254	·252	·272	·925	− 113	1·011
− 119	0				·0018											
− 114	0				·0016											
− 105	·363	2·170	·768	·177	·0354	·0342	3·50	·0506	·032	5·02	·255	·253	·273	·925	− 99	1·003
− 103	0				·0009											
− 90	·361	2·170	·760	·174	·0334	·0331	3·39	·0514	·033	5·02	·258	·256	·273	·940	− 86	1·013
− 85	0				− ·0002											
− 72	·366	2·210	·786	·180	·0342	·0344	3·52	·0512	·033	5·02	·257	·255	·273	·936	− 69	1·000
− 69	0				− ·0002											
− 55	·368	2·230	·797	·182	·0343	·0345	3·53	·0516	·034	5·02	·259	·257	·273	·943	− 53	·999
− 50	0				− ·0002											
− 39	·368	2·230	·797	·182	·0344	·0344	3·52	·0517	·034	5·02	·259	·257	·273	·943	− 38	·991
− 35	0				·0002											
− 22	·368	2·240	·801	·183	·0347	·0343	3·51	·0522	·035	5·02	·262	·260	·273	·954	− 21	·993
− 21	0				·0006											
− 12	·367	2·250	·802	·183	·0354	·0342	3·50	·0523	·036	5·02	·262	·260	·274	·951	− 12	·985
− 7	0				·0018											
+ 1	·368	2·250	·805	·184	·0358	·0338	3·46	·0532	·036	5·02	·267	·265	·274	·969	+ 1	·996
2	0				·0021											
10	·366	2·254	·802	·183	·0360	·0337	3·45	·0531	·037	5·03	·267	·265	·275	·965	10	·989
11	0				·0024											
18	·367	2·260	·806	·183	·0366	·0341	3·49	·0525	·037	5·03	·264	·262	·275	·955	18	·973
20	0				·0027											
27	·367	2·260	·806	·183	·0369	·0341	3·49	·0525	·037	5·03	·264	·261	·275	·951	27	·964
26	0				·0029											
32	·366	2·264	·805	·183	·0370	·0339	3·47	·0528	·037	5·03	·265	·262	·275	·955	32	·965
31	0				·0032											

Duration of experiment, 5 hours 10 minutes.

* See note on p. 407.

ZINC ROD.

Turned from a Cast Stick of "Pure redistilled Zinc." Fracture crystalline, with Crystals radiating from Axis of Rod. Density at 21° C. = 7·10.

$a_2 = 4{\cdot}10$ centims., approx. $k = {\cdot}26$, therefore $b' = {\cdot}29$, $x_A = {\cdot}39$, $x_B = 5{\cdot}07$, $x_C = 5{\cdot}46$, $x_{C'} = 5{\cdot}65$ centims. $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat / temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 21	0				+ ·0032											
24	·205	1·236	·248	·0565	·0375	·0342	3·50	·0161	·073	4·80	·0773	·0749	·290	·258	+ 24	·262
22	0				·0032											
− 199	0				− ·0027											
− 188	·211	1·200	·249	·0575	·0354	·0372	3·81	·0151	·057	4·75	·0718	·0704	·281	·251	− 174	·283
− 184	0				− ·0009											
− 174	·212	1·210	·252	·0582	·0370	·0374	3·83	·0152	·057	4·75	·0723	·0708	·281	·252	− 161	·283
− 174	0				·0000											
− 163	·212	1·215	·253	·0582	·0359	·0373	3·82	·0152	·058	4·75	·0722	·0707	·282	·251	− 150	·280
− 162	0				− ·0020											
− 155	·212	1·220	·254	·0584	·0359	·0379	3·88	·0150	·059	4·76	·0715	·0699	·282	·248	− 144	·275
− 152	0				− ·0020											
− 144	·212	1·230	·256	·0589	·0363	·0382	3·91	·0150	·060	4·76	·0715	·0699	·283	·247	− 134	·273
− 141	0				− ·0019											
− 133	·212	1·230	·256	·0589	·0368	·0384	3·93	·0149	·061	4·76	·0710	·0693	·283	·245	− 124	·270
− 132	0				− ·0014											
− 123	·212	1·231	·256	·0589	·0366	·0384	3·93	·0149	·062	4·76	·0710	·0693	·284	·244	− 115	·267
− 122	0				− ·0023											
− 113	·212	1·235	·257	·0591	·0353	·0388	3·97	·0149	·062	4·76	·0710	·0693	·284	·244	− 107	·266
− 111	0				− ·0028											
− 101	0				− ·0055											
− 91	·208	1·225	·249	·0573	·0313	·0365	3·74	·0153	·063	4·77	·0730	·0712	·284	·251	− 87	·271
− 84	0				− ·0048											
− 73	·207	1·230	·250	·0575	·0313	·0359	3·67	·0157	·065	4·77	·0749	·0730	·285	·256	− 69	·274
− 70	0				− ·0044											
− 61	·207	1·230	·250	·0572	·0313	·0357	3·65	·0157	·066	4·77	·0749	·0730	·286	·255	− 59	·271
− 54	0				− ·0042											
− 40	·207	1·230	·250	·0572	·0317	·0354	3·62	·0158	·068	4·78	·0756	·0736	·287	·256	− 39	·269
− 35	0				− ·0032											
− 23	·207	1·235	·251	·0575	·0321	·0349	3·57	·0161	·069	4·79	·0772	·0750	·288	·261	− 22	·272
− 18	0				− ·0025											
− 9	·206	1·240	·250	·0572	·0326	·0345	3·53	·0162	·070	4·79	·0777	·0755	·288	·262	− 9	·271
− 5	0				− ·0012											
+ 4	·206	1·240	·250	·0572	·0342	·0348	3·56	·0161	·071	4·79	·0772	·0749	·289	·259	+ 4	·266
7	0				·0000											
15	·206	1·242	·250	·0570	·0347	·0343	3·51	·0162	·072	4·80	·0778	·0755	·289	·261	15	·267
16	0				·0009											
23	·206	1·245	·251	·0572	·0355	·0343	3·51	·0163	·073	4·80	·0783	·0759	·290	·262	23	·266
23	0				·0015											

Duration of experiment, 6 hours 20 minutes.

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* See note on p. 407.

ZINC ROD (Second Experiment).

$a_2 = 2{\cdot}32$ centims., approx. $k = {\cdot}26$, therefore $b' = {\cdot}29$, $x_A = {\cdot}39$, $x_B = 3{\cdot}29$, $x_C = 3{\cdot}68$, $x_{C'} = 3{\cdot}87$ centims. ; $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 19	0				+ ·0032											
24	·270	1·640	·433	·0987	·0413	·0381	3·90	·0253	·073	2·93	·0741	·0725	·278	·261	+ 24	·265
21	0				·0032											
− 195	0				− ·0044											
− 182	·279	1·610	·439	·1014	·0384	·0427	4·37	·0232	·057	2·92	·0678	·0668	·274	·244	− 169	·275
− 179	0				− ·0042											
− 167	·280	1·626	·446	·1024	·0395	·0435	4·45	·0231	·058	2·92	·0675	·0665	·274	·243	− 155	·272
− 164	0				− ·0037											
− 153	·282	1·640	·453	·1042	·0404	·0439	4·49	·0232	·059	2·92	·0678	·0667	·275	·247	− 142	·275
− 152	0				− ·0034											
− 141	·282	1·650	·455	·1047	·0412	·0444	4·54	·0230	·060	2·92	·0672	·0661	·275	·240	− 132	·265
− 141	0				− ·0029											
− 131	·282	1·660	·458	·1054	·0415	·0442	4·52	·0233	·061	2·92	·0681	·0670	·275	·244	− 123	·269
− 129	0				− ·0025											
− 121	·282	1·660	·458	·1054	·0419	·0442	4·52	·0233	·062	2·92	·0681	·0670	·275	·244	− 114	·267
− 121	0				− ·0020											
− 114	0				− ·0031											
− 102	·285	1·700	·475	·1093	·0419	·0447	4·58	·0238	·063	2·92	·0696	·0684	·275	·249	− 97	·271
− 98	0				− ·0025											
− 87	·280	1·680	·461	·1060	·0412	·0434	4·44	·0238	·063	2·92	·0696	·0684	·275	·249	− 83	·268
− 84	0				− ·0019											
− 68	·284	1·710	·475	·1088	·0424	·0440	4·50	·0241	·065	2·92	·0704	·0691	·276	·250	− 65	·267
− 65	0				− ·0013											
− 51	·282	1·710	·472	·1081	·0423	·0437	4·47	·0241	·067	2·93	·0707	·0694	·276	·251	− 49	·265
− 46	0				− ·0014											
− 34	·282	1·710	·472	·1081	·0424	·0431	4·41	·0245	·068	2·93	·0718	·0704	·276	·255	− 33	·267
− 31	0				0000											
− 20	·283	1·730	·479	·1097	·0427	·0427	4·37	·0251	·069	2·93	·0735	·0720	·276	·261	− 19	·272
− 18	0				0000											
− 7	·283	1·734	·480	·1099	·0427	·0426	4·36	·0252	·070	2·93	·0738	·0723	·277	·261	− 7	·270
− 6	0				·0002											
+ 4	·283	1·740	·483	·1106	·0433	·0424	4·34	·0255	·071	2·93	·0747	·0732	·277	·265	+ 4	·272
4	0				·0019											
11	·282	1·740	·481	·1100	·0436	·0418	4·28	·0257	·072	2·93	·0753	·0738	·277	·267	11	·273
12	0				·0017											
20	·282	1·740	·481	·1097	·0437	·0418	4·28	·0256	·073	2·93	·0750	·0734	·278	·264	20	·269
20	0				·0021											
27	·282	1·744	·481	·1097	·0440	·0418	4·28	·0256	·073	2·93	·0750	·0734	·278	·264	27	·268
26	0				·0023											

Duration of experiment, 4 hours.

* See note on p. 407.

CADMIUM ROD.

Turned from a Cast Stick of "Pure Redistilled Cadmium" as used in Cadmium Cell. Density at 21° C. = 8·64.

$a_2 = 4{\cdot}32$ centims., approx. $k = {\cdot}217$, therefore $b' = {\cdot}29$, $x_A = {\cdot}39$, $x_B = 5{\cdot}29$, $x_C = 5{\cdot}68$, $x_{C'} = 5{\cdot}87$ centims.,
$s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$ ohms.	Differences, ohms.	Pt. temp. differences.	$\frac{\text{Heat}}{\text{temp difference}}$.	$\sqrt{6{\cdot}85\,\frac{h}{k}}$.	$x_B S_B - x_A S_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 20	0				+ ·0029											
24	·219	1·330	·286	·0651	·0520	·0493	5·05	·0129	·079	5·06	·0655	·0630	·296	·213	+ 24	·216
21	0				·0026											
− 203	0				− ·0015											
− 191	·222	1·270	·277	·0639	·0486	·0513	5·25	·0121	·062	5·00	·0607	·0592	·285	·208	− 177	·235
− 177	0				− ·0039											
− 165	·219	1·264	·272	·0625	·0454	·0486	4·97	·0125	·063	5·00	·0627	·0611	·286	·213	− 153	·238
− 161	0				− ·0025											
− 150	·219	1·262	·272	·0625	·0461	·0481	4·92	·0127	·065	5·00	·0637	·0621	·287	·216	− 140	·239
− 147	0				− ·0015											
− 138	·220	1·270	·274	·0629	·0470	·0483	4·94	·0127	·066	5·01	·0638	·0620	·287	·216	− 129	·238
− 135	0				− ·0012											
− 126	·219	1·272	·274	·0629	·0476	·0486	4·97	·0126	·067	5·01	·0633	·0615	·288	·213	− 118	·234
− 125	0				− ·0008											
− 116	·218	1·272	·273	·0627	·0475	·0486	4·97	·0126	·068	5·02	·0635	·0617	·289	·213	− 109	·233
− 115	0				− ·0013											
− 105	·218	1·280	·274	·0629	·0462	·0484	4·95	·0127	·069	5·02	·0640	·0622	·289	·215	− 99	·234
− 101	0				− ·0031											
− 90	·218	1·279	·274	·0629	·0455	·0485	4·96	·0126	·070	5·02	·0635	·0616	·290	·212	− 86	·229
− 86	0				− ·0030											
− 76	0				− ·0052											
− 63	·217	1·290	·275	·0629	·0433	·0480	4·91	·0128	·072	5·03	·0646	·0626	·291	·215	− 60	·229
− 55	0				− ·0042											
− 43	·215	1·279	·270	·0617	·0434	·0472	4·83	·0127	·073	5·04	·0642	·0631	·291	·216	− 41	·227
− 36	0				− ·0034											
− 25	·214	1·280	·269	·0615	·0440	·0472	4·81	·0128	·075	5·04	·0647	·0624	·293	·213	− 24	·222
− 20	0				− ·0027											
− 10	·214	1·290	·271	·0619	·0041	·0463	4·74	·0130	·077	5·05	·0659	·0636	·294	·216	− 9	·224
− 6	0				− ·0018											
+ 3	·215	1·300	·274	·0626	·0459	·0468	4·79	·0130	·078	5·05	·0659	·0635	·294	·216	+ 3	·222
5	0				·0000											
13	·214	1·292	·271	·0619	·0463	·0463	4·74	·0130	·079	5·06	·0660	·0636	·295	·215	13	·220
15	0				·0000											
21	·214	1·295	·272	·0619	·0472	·0468	4·79	·0129	·079	5·06	·0655	·0630	·295	·213	21	·217
21	0				·0007											

Duration of experiment, 7 hours 20 minutes.

* See note on p. 407.

CADMIUM ROD (Second Experiment).

$a_2 = 2{\cdot}34$ centims., approx. $k = {\cdot}217$, therefore $b' = {\cdot}29$, $x_A = {\cdot}39$, $x_B = 3{\cdot}31$, $x_C = 3{\cdot}70$, $x_{C'} = 3{\cdot}89$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H temp. of rod.	k_H.
° C.															° C.	
+ 20	0				+ ·0044											
22	·213	1·300	·272	·0619	·0339	·0296	3·03	·0205	·079	2·96	·0604	·0588	·280	·210	+ 22	·213
20	0				·0042											
− 203	0				− ·0034											
− 181	·217	1·250	·268	·0618	·0260	·0295	3·02	·0205	·062	2·94	·0603	·0593	·275	·216	− 168	·243
− 177	0				− ·0035											
− 165	·217	1·260	·270	·0620	·0265	·0298	3·05	·0204	·063	2·94	·0600	·0589	·275	·214	− 153	·238
− 161	0				− ·0032											
− 151	·217	1·260	·270	·0620	·0272	·0301	3·08	·0202	·065	2·94	·0596	·0585	·276	·212	− 141	·235
− 149	0				− ·0026											
− 140	·216	1·265	·269	·0618	·0277	·0301	3·08	·0201	·066	2·95	·0593	·0582	·276	·211	− 131	·233
− 139	0				− ·0022											
− 133	0				− ·0028											
− 122	·220	1·314	·284	·0652	·0288	·0313	3·20	·0204	·067	2·95	·0602	·0591	·276	·214	− 115	·234
− 120	0				− ·0022											
− 110	·219	1·320	·284	·0652	·0293	·0313	3·20	·0204	·068	2·95	·0602	·0590	·276	·214	− 104	·233
− 107	0				− ·0019											
− 96	·219	1·320	·284	·0652	·0297	·0313	3·20	·0204	·069	2·95	·0602	·0590	·277	·213	− 91	·230
− 94	0				− ·0014											
− 83	·219	1·330	·286	·0657	·0303	·0314	3·21	·0205	·070	2·95	·0605	·0593	·277	·214	− 79	·230
− 80	0				− ·0009											
− 71	·217	1·320	·281	·0642	·0303	·0310	3·17	·0203	·071	2·95	·0599	·0586	·277	·212	− 68	·226
− 68	0				− ·0005											
− 62	0				− ·0017											
− 50	·217	1·312	·279	·0638	·0295	·0308	3·15	·0203	·072	2·95	·0599	·0586	·278	·211	− 48	·223
− 46	0				− ·0009											
− 36	·217	1·320	·281	·0642	·0302	·0308	3·15	·0204	·074	2·95	·0602	·0588	·278	·211	− 35	·221
− 33	0				− ·0004											
− 22	·217	1·314	·279	·0638	·0310	·0309	3·16	·0202	·075	2·95	·0596	·0582	·279	·209	− 21	·217
− 19	0				·0006											
− 10	·217	1·320	·281	·0642	·0314	·0306	3·13	·0206	·077	2·95	·0608	·0593	·279	·212	− 10	·219
− 6	0				·0010											
+ 2	·217	1·320	·281	·0642	·0314	·0303	3·10	·0208	·078	2·95	·0616	·0601	·280	·215	+ 2	·221
3	0				·0013											
10	·216	1·322	·280	·0640	·0321	·0306	3·13	·0205	·079	2·96	·0607	·0591	·280	·211	10	·216
12	0				·0017											
19	·216	1·322	·280	·0637	·0324	·0304	3·11	·0205	·079	2·96	·0607	·0591	·280	·210	19	·214
20	0				·0022											

Duration of experiment, 5 hours.

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* See note on p. 407.

ALUMINIUM ROD.

Turned from a Rod supplied by Messrs. JOHNSON and MATTHEY as ·99 Al. Density at 20° C. = 2·70.

$a_2 = 4{\cdot}38$ centims., approx. $k = {\cdot}48$, therefore $b' = {\cdot}31$, $x_A = {\cdot}41$, $x_B = 5{\cdot}41$, $x_C = 5{\cdot}72$, $x_{C'} = 5{\cdot}93$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\,\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 17	0				+ ·0034											
19	·250	1·52	·372	·0849	·0335	·0300	3·07	·0277	·053	5·08	·141	·138	·281	·491	+ 19	·500
19	0				·0035											
− 195	0				·0000											
− 179	·258	1·48	·373	·0862	·0362	·0322	3·30	·0261	·044	5·05	·132	·130	·276	·471	− 166	·528
− 176	0				·0048											
− 166	·257	1·48	·374	·0861	·0388	·0333	3·41	·0252	·044	5·05	·127	·125	·276	·453	− 154	·508
− 164	0				·0063											
− 155	0				·0009											
− 146	·257	1·49	·374	·0861	·0350	·0339	3·47	·0248	·045	5·05	·125	·123	·277	·444	− 136	·492
− 146	0				·0013											
− 137	·257	1·49	·374	·0861	·0354	·0339	3·47	·0248	·046	5·05	·125	·123	·277	·444	− 128	·490
− 136	0				·0017											
− 130	·257	1·50	·377	·0868	·0357	·0339	3·47	·0250	·046	5·05	·126	·124	·277	·448	− 122	·492
− 127	0				·0019											
− 121	·256	1·50	·375	·0863	·0358	·0339	3·47	·0248	·046	5·05	·125	·123	·277	·444	− 114	·486
− 121	0				·0020											
− 116	0				·0000											
− 106	·262	1·56	·400	·0920	·0344	·0349	3·57	·0258	·047	5·05	·130	·128	·278	·461	− 100	·501
− 104	0				− ·0010											
− 93	·263	1·58	·407	·0936	·0343	·0353	3·61	·0259	·047	5·05	·131	·129	·278	·464	− 88	·501
− 90	0				− ·0010											
− 77	·263	1·58	·407	·0936	·0339	·0349	3·57	·0262	·048	5·06	·133	·131	·278	·472	− 73	·496
− 75	0				− ·0009											
− 66	·263	1·59	·409	·0936	·0339	·0346	3·54	·0264	·049	5·06	·134	·132	·279	·474	− 63	·497
− 63	0				− ·0006											
− 53	0				− ·0017											
− 40	·261	1·576	·402	·0920	·0328	·0339	3·47	·0265	·050	5·06	·134	·132	·280	·472	− 39	·496
− 34	0				− ·0006											
− 23	·260	1·58	·401	·0918	·0325	·0330	3·38	·0272	·050	5·07	·138	·136	·280	·486	− 22	·507
− 19	0				− ·0004											
− 8	·259	1·58	·401	·0918	·0324	·0326	3·34	·0275	·051	5·07	·139	·137	·280	·490	− 8	·507
− 4	0				·0000											
+ 6	·259	1·59	·402	·0920	·0335	·0329	3·37	·0273	·052	5·08	·139	·137	·280	·490	+ 6	·503
9	0				·0013											
17	·259	1·59	·402	·0920	·0337	·0323	3·31	·0278	·053	5·08	·141	·139	·280	·497	17	·508
17	0				·0017											
24	·259	1·59	·403	·0919	·0339	·0320	3·28	·0280	·053	5·08	·142	·139	·280	·497	24	·505
25	0				·0021											

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* See note on p. 407.

TIN ROD.

Turned from a Bar of Pure Tin supplied by Messrs. KAHLBAUM. Density at 21° C. = 7·28.

$a_2 = 4·50$ centims., approx. $k = ·16$, therefore $b' = ·27$, $x_A = ·37$, $x_B = 5·41$, $x_C = 5·78$, $x_{C'} = 5·98$ centims.; $s = 1·83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	$\frac{\text{Heat}}{\text{temp. difference}}$.	$\sqrt{6·85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 23	0				+ ·0034											
27	·183	1·100	·197	·0449	·0493	·0460	4·71	·00953	·093	5·27	·0503	·0477	·308	·155	+ 27	·157
25	0				·0032											
− 200	0				− ·0049											
− 186	·185	1·057	·192	·0445	·0380	·0428	4·38	·01020	·073	5·18	·0529	·0513	·292	·175	− 174	·197
− 179	0				− ·0047											
− 169	·185	1·060	·193	·0445	·0396	·0439	4·49	·00990	·074	5·19	·0514	·0498	·293	·170	− 157	·190
− 166	0				− ·0040											
− 153	·185	1·061	·193	·0444	·0407	·0442	4·52	·00984	·075	5·19	·0511	·0494	·293	·168	− 143	·187
− 135	0				− ·0027											
− 126	·185	1·061	·193	·0444	·0419	·0448	4·59	·00968	·078	5·20	·0504	·0486	·296	·164	− 118	·180
− 124	0				− ·0030											
− 116	0				− ·0064											
− 106	·186	1·097	·200	·0460	·0400	·0465	4·76	·00967	·080	5·21	·0504	·0485	·297	·163	− 110	·178
− 96	0				− ·0066											
− 86	·187	1·104	·203	·0466	·0412	·0475	4·86	·00959	·082	5·22	·0501	·0481	·298	·161	− 82	·173
− 79	0				− ·0060											
− 68	·187	1·110	·204	·0467	·0419	·0477	4·88	·00957	·083	5·22	·0500	·0480	·299	·160	− 65	·171
− 62	0				− ·0056											
− 47	·187	1·117	·205	·0469	·0428	·0481	4·92	·00953	·085	5·23	·0499	·0478	·301	·159	− 45	·168
− 42	0				− ·0050											
− 38	·187	1·121	·206	·0471	·0436	·0481	4·92	·00957	·087	5·24	·0502	·0479	·302	·158	− 30	·165
− 24	0				− ·0041											
− 18	0				− ·0066											
− 8	·185	1·111	·202	·0462	·0421	·0477	4·88	·00947	·089	5·25	·0497	·0473	·304	·155	− 8	·160
+ 1	0				− ·0045											
12	·185	1·120	·203	·0462	·0444	·0477	4·88	·00947	·092	5·27	·0499	·0474	·307	·154	+ 14	·157
22	0				− ·0022											
30	·186	1·130	·205	·0467	·0459	·0477	4·88	·00947	·093	5·27	·0499	·0473	·308	·153	30	·155
34	0				− ·0013											

Duration of experiment, 5 hours 20 minutes.

* See note on p. 407.

LEAD ROD.

Turned from a Bar of Pure Lead supplied by Messrs. BAXENDALE, Manchester. Density at 25° C. = 11·29.

$a_2 = 1{\cdot}90$ centims., approx. $k = {\cdot}082$, therefore $b' = {\cdot}23$, $x_A = {\cdot}33$, $x_B = 2{\cdot}69$, $x_C = 3{\cdot}02$, $x_{C'} = 3{\cdot}17$ centims. $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	$\frac{\text{Heat}}{\text{temp. difference}}$.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 18	0				+ ·0031											
23	·202	1·225	·242	·0552	·0558	·0526	5·38	·01026	·130	2·42	·0249	·0235	·289	·0814	+ 23	·0828
20	0				·0032											
− 194	0				− ·0064											
− 177	·209	1·200	·246	·0568	·0499	·0559	5·72	·00993	·102	2·39	·0238	·0229	·281	·0816	− 164	·0926
− 179	0				− ·0058											
− 151	·210	1·220	·251	·0577	·0514	·0568	5·81	·00993	·105	2·39	·0238	·0229	·282	·0813	− 141	·0902
− 145	0				− ·0050											
− 129	·210	1·225	·252	·0579	·0527	·0573	5·86	·00988	·109	2·40	·0237	·0227	·283	·0803	− 121	·0882
− 124	0				− ·0042											
− 110	·210	1·230	·253	·0582	·0542	·0580	5·93	·00982	·110	2·40	·0235	·0225	·283	·0796	− 104	·0867
− 107	0				− ·0033											
− 97	·210	1·235	·254	·0584	·0547	·0578	5·92	·00986	·113	2·40	·0237	·0227	·284	·0800	− 92	·0866
− 94	0				− ·0029											
− 82	·210	1·240	·255	·0586	·0555	·0580	5·94	·00987	·115	2·40	·0237	·0226	·285	·0794	− 78	·0853
− 81	0				− ·0021											
− 64	·210	1·245	·257	·0588	·0562	·0580	5·94	·00990	·117	2·40	·0237	·0226	·285	·0794	− 61	·0847
− 64	0				− ·0014											
− 54	·210	1·250	·257	·0588	·0567	·0581	5·95	·00988	·118	2·41	·0238	·0227	·286	·0794	− 52	·0841
− 54	0				− ·0015											
− 48	0				− ·0017											
− 38	·213	1·290	·270	·0618	·0582	·0600	6·14	·01006	·120	2·41	·0243	·0231	·286	·0808	− 37	·0850
− 37	0				− ·0018											
− 26	·212	1·290	·268	·0614	·0582	·0598	6·12	·01003	·122	2·41	·0243	·0231	·287	·0805	− 25	·0841
− 26	0				− ·0014											
− 18	·212	1·290	·268	·0614	·0571	·0588	6·02	·01020	·124	2·41	·0245	·0233	·287	·0812	− 17	·0844
− 18	0				− ·0021											
− 12	0				− ·0021											
− 1	·208	1·264	·258	·0591	·0549	·0563	5·76	·01026	·126	2·41	·0248	·0235	·288	·0816	− 1	·0841
+ 2	0				− ·0008											
11	·207	1·260	·256	·0586	·0554	·0558	5·71	·01026	·128	2·41	·0248	·0235	·289	·0814	+ 11	·0832
11	0				·0000											
19	·2055	1·254	·252	·0577	·0550	·0547	5·60	·01030	·129	2·42	·0249	·0236	·289	·0817	19	·0834
19	0				·0006											
26	·205	1·250	·251	·0574	·0548	·0540	5·53	·01038	·130	2·42	·0249	·0235	·289	·0814	26	·0828
24	0				·0010											

Duration of experiment, 5 hours 30 minutes.

* See note on p. 407.

WROUGHT IRON ROD.

Turned from a Bar of "Best Scrap Iron." Fe = ·9942, C = ·001, Mn = ·0015, Si = ·0013.*
Density at 21° C. = 7·74.

$a_2 = 3{\cdot}80$ centims., approx. $k = {\cdot}14$, therefore $b' = {\cdot}27$, $x_A = {\cdot}37$, $x_B = 4{\cdot}71$, $x_C = 5{\cdot}08$, $x_{C'} = 5{\cdot}26$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.†	II.†	III.†	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
−184	0				−·0034											
−172	·173	·990	·169	·0391	·0401	·0422	4·32	·00905	·079	4·45	·0403	·0389	·290	·134	−160	·150
−158	0				−·0007											
−151	·173	·990	·169	·0389	·0341	·0398	4·07	·00956	·081	4·45	·0425	·0410	·291	·141	−140	·156
−144	0				−·0081											
−133	·173	·997	·170	·0392	·0321	·0404	4·13	·00949	·083	4·46	·0424	·0408	·292	·139	−125	·153
−127	0				−·0084											
−116	·171	·990	·167	·0385	·0310	·0392	4·01	·00960	·084	4·47	·0430	·0413	·293	·140	−109	·153
−110	0				−·0079											
−100	·170	·990	·166	·0382	·0313	·0388	3·97	·00962	·086	4·47	·0430	·0413	·294	·140	− 95	·151
− 96	0				−·0071											
− 72	0				−·0056											
− 63	·172	1·012	·171	·0392	·0340	·0398	4·07	·00963	·090	4·48	·0432	·0413	·297	·139	− 60	·148
− 54	0				−·0059											
− 46	·170	1·005	·168	·0385	·0334	·0388	3·97	·00970	·092	4·49	·0436	·0417	·298	·139	− 44	·146
− 44	0				−·0048											
− 17	0				−·0085											
− 1	·172	1·030	·174	·0399	·0319	·0388	3·97	·0101	·097	4·50	·0455	·0434	·301	·144	− 1	·148
+ 2	0				−·0053											
13	·172	1·035	·175	·0400	·0343	·0387	3·96	·0101	·098	4·51	·0456	·0434	·302	·143	+ 13	·146
16	0				−·0036											

Duration of experiment, 5 hours.

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* I have to thank my colleague, Professor J. T. HEWITT, for the analysis of this specimen.

† See note on p. 407.

NICKEL ROD.

Turned from a Bar supplied by Messrs. JOHNSON and MATTHEY as ·99 Ni. Density at 21° C. = 8·80.

a_2=3·86 centims., approx. k=·14, therefore b'=·27, x_A=·37, x_B=4·77, x_C=5·14, $x_{C'}$=·32 centims., s=1·83 sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	R_B-R_A, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6\cdot85\frac{h}{k}}$.	$x_BS_B-x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
°C.															°C.	
+ 28	·190	1·150	·215	·0490	+ ·0542	·0507	5·19	·00944	·099	4·58	·0432	·0409	·303	·136	+ 28	·138
25	0				+ ·0035											
− 183	0				− ·0100											
− 170	·191	1·096	·206	·0475	·0504	·0598	6·12	·00775	·085	4·53	·0352	·0338	·294	·115	− 159	·129
− 163	0				− ·0089											
− 150	·192	1·101	·208	·0478	·0500	·0582	5·96	·00802	·088	4·54	·0365	·0350	·296	·116	− 143	·131
− 146	0				− ·0076											
− 135	·192	1·110	·209	·0480	·0497	·0574	5·87	·00817	·089	4·54	·0371	·0355	·296	·120	− 126	·132
− 132	0				− ·0078											
− 120	·1916	1·110	·209	·0480	·0499	·0583	5·97	·00803	·091	4·55	·0366	·0349	·297	·118	− 113	·129
− 116	0				− ·0090											
− 102	·191	1·116	·208	·0478	·0457	·0546	5·59	·00854	·086	4·53	·0387	·0370	·294	·126	− 97	·136
− 98	0				− ·0088											
− 84	·191	1·120	·210	·0483	·0456	·0540	5·53	·00873	·087	4·53	·0396	·0378	·296	·128	− 80	·138
− 80	0				− ·0080											
− 68	·191	1·127	·212	·0485	·0459	·0539	5·52	·00878	·089	4·54	·0399	·0381	·297	·128	− 65	·137
− 64	0				− ·0079											
− 50	0				− ·0113											
− 31	·192	1·150	·217	·0496	·0437	·0531	5·44	·00912	·093	4·56	·0416	·0396	·299	·133	− 30	·139
− 28	0				− ·0084											
− 15	·191	1·156	·217	·0496	·0448	·0522	5·34	·00930	·094	4·56	·0424	·0403	·299	·136	− 15	·140
− 13	0				− ·0064											
− 1	·191	1·160	·218	·0498	·0466	·0516	5·28	·00943	·096	4·57	·0431	·0409	·301	·137	− 1	·141
+ 2	0				− ·0038											
11	·191	1·160	·218	·0498	·0481	·0511	5·23	·00954	·098	4·58	·0437	·0415	·302	·138	+ 11	·141
14	0				− ·0022											

Duration of experiment, 5 hours 30 minutes.

* See note on p. 407.

STEEL ROD.

Turned from a Bar of High Carbon Steel, known as "Silver Steel," containing approximately 1 per cent. of Carbon.
Density at 24° C. = 7·84.

$a_2 = 4{\cdot}30$ centims., approx. $k = {\cdot}11$, therefore $b' = {\cdot}26$, $x_A = {\cdot}36$, $x_B = 5{\cdot}18$, $x_C = 5{\cdot}44$, $x_{C'} = 5{\cdot}61$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	$\frac{\text{Heat}}{\text{temp. difference.}}$	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 23	·170	1·03	·172	·0393	+ ·0550	·0523	5·36	·00734	·111	5·13	·0377	·0356	·319	·112	+ 23	·114
19	0				+ ·0027											
− 131	0				− ·0035											
− 114	·169	·99	·165	·0380	·0485	·0541	5·54	·00686	·095	5·03	·0345	·0329	·304	·108	− 107	·117
− 117	0				− ·0077											
− 96	·168	·99	·164	·0378	·0470	·0545	5·58	·00678	·096	5·04	·0342	·0325	·306	·106	− 91	·114
− 98	0				− ·0073											
− 79	·169	1·00	·167	·0385	·0468	·0540	5·53	·00696	·099	5·05	·0352	·0334	·308	·109	− 75	·116
− 81	0				− ·0070											
− 64	·171	1·01	·171	·0394	·0470	·0533	5·46	·00722	·100	5·06	·0365	·0347	·309	·112	− 61	·119
− 42	0				− ·0058											
− 28	·168	1·00	·166	·0381	·0460	·0508	5·20	·00733	·105	5·07	·0372	·0352	·314	·112	− 27	·117
− 31	0				− ·0042											
− 13	·167	·98	·162	·0372	·0460	·0494	5·06	·00736	·106	5·08	·0374	·0353	·315	·112	− 13	·116
− 18	0				− ·0029											
− 3	·167	·99	·162	·0372	·0472	·0494	5·06	·00736	·108	5·09	·0375	·0354	·315	·113	− 3	·115
− 9	0				− ·0015											
+ 4	·167	·99	·162	·0372	·0483	·0493	5·05	·00738	·109	5·10	·0377	·0356	·317	·112	+ 4	·115
− 3	0				− ·0005											
+ 9	·168	1·00	·164	·0374	·0487	·0490	5·02	·00746	·110	5·10	·0381	·0360	·318	·112	+ 9	·116
+ 2	0				·0000											
− 199	0				− ·0012											
− 184	·169	·96	·160	·0370	·0541	·0572	5·85	·00632	·088	4·99	·0315	·0301	·299	·101	− 168	·113
− 185	0				− ·0042											
− 187	0				− ·0109											
− 150	·168	·96	·158	·0364	·0419	·0554	5·67	·00642	·090	5·01	·0322	·0407	·301	·102	− 140	·113
− 145	0				− ·0148											
− 120	·169	·98	·162	·0373	·0414	·0554	5·67	·00658	·094	5·03	·0331	·0315	·304	·104	− 113	·112
− 118	0				− ·0132											
− 96	·170	·99	·164	·0378	·0413	·0548	5·61	·00674	·097	5·04	·0340	·0323	·306	·106	− 91	·114
− 95	0				− ·0139											
− 75	·170	·99	·164	·0378	·0415	·0538	5·51	·00686	·099	5·15	·0347	·0329	·308	·107	− 71	·114
− 75	0				− ·0107											

Duration of experiment, 1 hour + 6 hours + 6 hours.

* See note on p. 407.

BRASS ROD.

Turned from a Rod of Composition ·70 Cu, ·30 Zn, approx. Density at 22° C. = 8·44.

a_2 = 4·11 centims., approx. k = ·16 to ·26, therefore b' = ·282 to ·290, taken = ·285, x_A = ·39, x_B = 5·07, x_C = 5·45, $x_{C'}$ = 5·64 centims., s = 1·83 sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6\cdot85\,\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 20	0				+ ·0033											
23	·202	1·220	·241	·0549	·0368	·0336	3·44	·0160	·073	4·80	·0769	·0745	·289	·258	+ 23	·262
21	0				·0031											
− 189	0				− ·0059											
− 178	·208	1·190	·243	·0561	·0496	·0559	5·72	·0098	·073	4·80	·0471	·0456	·289	·158	− 165	·178
− 174	0				− ·0066											
− 161	·208	1·200	·245	·0563	·0472	·0529	5·41	·0104	·073	4·80	·0498	·0483	·289	·167	− 150	·186
− 157	0				− ·0048											
− 148	·209	1·210	·248	·0570	·0476	·0518	5·30	·0108	·073	4·80	·0517	·0501	·289	·174	− 138	·193
− 145	0				− ·0036											
− 139	0				− ·0061											
− 129	·214	1·265	·266	·0612	·0463	·0528	5·40	·0113	·072	4·80	·0540	·0523	·289	·181	− 121	·199
− 123	0				− ·0068											
− 109	·215	1·280	·270	·0621	·0428	·0496	5·08	·0123	·072	4·80	·0583	·0567	·289	·196	− 103	·212
− 103	0				− ·0069											
− 89	·215	1·282	·271	·0623	·0413	·0477	4·88	·0127	·072	4·80	·0612	·0594	·289	·206	− 85	·221
− 83	0				− ·0059											
− 72	·215	1·290	·272	·0623	·0405	·0459	4·70	·0132	·072	4·80	·0635	·0616	·289	·213	− 69	·228
− 66	0				− ·0049											
− 52	0				− ·0093											
− 36	·212	1·270	·264	·0604	·0334	·0414	4·24	·0142	·072	4·80	·0683	·0662	·289	·229	− 35	·240
− 27	0				− ·0067											
− 13	·212	1·272	·264	·0604	·0332	·0390	3·99	·0151	·072	4·80	·0725	·0703	·289	·243	− 13	·252
− 4	0				− ·0049											
+ 8	·211	1·280	·266	·0609	·0346	·0385	3·94	·0154	·072	4·80	·0740	·0717	·289	·248	+ 8	·253
14	0				− ·0029											
25	·211	1·282	·266	·0606	·0346	·0369	3·78	·0160	·073	4·80	·0769	·0745	·289	·257	25	·261
28	0				− ·0018											

Duration of experiment, 5 hours.

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* See note on p. 407.

GERMAN SILVER ROD.

Turned from a Rod supplied by Messrs. JOHNSON and MATTHEY of Composition ·62 Cu, ·15 Ni, ·22 Zn, approx.
Density at 22° C. = 8·67.

$a_2 = 4{\cdot}26$ centims., approx. $k = {\cdot}04$ to ${\cdot}06$, therefore $b' = {\cdot}22$, $x_A = {\cdot}32$, $x_B = 5{\cdot}02$, $x_C = 5{\cdot}34$, $x_{C'} = 5{\cdot}49$ centims., $s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	$\frac{\text{Heat}}{\text{temp. difference.}}$	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 21	0				+ ·0031											
25	·142	·854	·119	·0272	·0614	·0582	5·96	·00456	·152	5·20	·0237	·0212	·361	·0588	+ 25	·0597
22	0				·0033											
− 195	0				− ·0148											
− 176	·146	·834	·120	·0278	·0773	·0934	9·56	·00291	·145	5·16	·0150	·0134	·351	·0382	− 163	·0428
− 165	0				− ·0172											
− 144	·146	·837	·120	·0276	·0704	·0855	8·76	·00318	·145	5·16	·0162	·0145	·351	·0413	− 134	·0456
− 142	0				− ·0133											
− 126	·146	·841	·121	·0279	·0716	·0878	8·99	·00310	·146	5·17	·0160	·0142	·353	·0402	− 118	·0440
− 120	0				− ·0191											
− 100	·146	·846	·122	·0281	·0606	·0793	8·12	·00346	·147	5·17	·0179	·0161	·354	·0454	− 95	·0493
− 90	0				− ·0185											
− 74	·146	·851	·122	·0281	·0591	·0769	7·87	·00357	·149	5·18	·0185	·0165	·358	·0460	− 70	·0493
− 63	0				− ·0172											
− 46	·146	·860	·124	·0284	·0578	·0728	7·45	·00381	·149	5·18	·0197	·0175	·358	·0490	− 44	·0517
− 40	0				− ·0128											
− 26	·146	·861	·124	·0284	·0588	·0692	7·08	·00401	·150	5·18	·0207	·0184	·358	·0515	− 25	·0538
− 22	0				− ·0085											
− 11	·146	·864	·124	·0284	·0597	·0662	6·78	·00419	·151	5·20	·0217	·0193	·361	·0536	− 11	·0554
− 10	0				− ·0046											
− 5	0				− ·0085											
+ 7	·148	·901	·131	·0300	·0614	·0676	6·92	·00432	·151	5·20	·0225	·0198	·361	·0549	+ 7	·0563
8	0				− ·0039											
16	·148	·901	·131	·0300	·0622	·0649	6·64	·00451	·150	5·18	·0234	·0208	·358	·0582	16	·0594
17	0				− ·0019											

Duration of experiment, 8 hours 20 minutes.

* See note on p. 407.

PLATINOID ROD.

Turned from a Rod supplied by the London Electric Wire Company. Composition approximately that of German Silver. Density at 22° C. = 8·66.

a_2 = 1·51 centims., approx. k = ·04 to ·06, therefore b' = ·21, x_A = ·31, x_B = 2·24, x_C = 2·55, $x_{C'}$ = 2·69 centims., s = 1·83 sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
− 186	0				− ·0134											
− 170	·181	1·040	·184	·0426	·0627	·0757	7·75	·00549	·150	1·97	·0109	·0102	·288	·0354	− 158	·0397
− 164	0				− ·0125											
− 149	·181	1·050	·186	·0428	·0605	·0722	7·39	·00579	·150	1·97	·0115	·0107	·288	·0371	− 139	·0412
− 145	0				− ·0110											
− 129	·181	1·054	·187	·0431	·0579	·0682	6·98	·00617	·150	1·97	·0122	·0114	·288	·0395	− 121	·0435
− 124	0				− ·0097											
− 108	·181	1·060	·188	·0433	·0564	·0654	6·69	·00646	·150	1·97	·0128	·0120	·288	·0416	− 102	·0453
− 103	0				− ·0082											
− 90	·181	1·064	·189	·0435	·0549	·0623	6·38	·00681	·150	1·97	·0135	·0126	·288	·0437	− 86	·0471
− 86	0				− ·0066											
− 79	0				− ·0088											
− 65	·183	1·096	·197	·0452	·0521	·0601	6·15	·00733	·150	1·97	·0145	·0136	·288	·0474	− 62	·0505
− 61	0				− ·0071											
− 45	·179	1·080	·190	·0436	·0488	·0550	5·63	·00774	·150	1·97	·0153	·0143	·288	·0498	− 43	·0525
− 30	0				− ·0054											
− 26	·179	1·086	·191	·0438	·0481	·0525	5·39	·00815	·150	1·97	·0161	·0151	·288	·0526	− 25	·0549
− 24	0				− ·0035											
− 12	·179	1·090	·191	·0438	·0470	·0498	5·10	·00858	·150	1·97	·0170	·0159	·288	·0553	− 12	·0575
− 10	0				− ·0021											
− 2	0				− ·0054											
+ 7	·177	1·070	·186	·0427	·0438	·0478	4·89	·00872	·150	1·97	·0172	·0161	·288	·0560	+ 7	·0575
10	0				− ·0025											
21	·176	1·070	·185	·0422	·0432	·0449	4·60	·00917	·150	1·97	·0181	·0170	·288	·0591	21	·0602
22	0				− ·0009											
30	·174	1·060	·181	·0413	·0426	·0428	4·38	·00942	·150	1·97	·0186	·0175	·288	·0609	31	·0616
31	0				+ ·0005											

Duration of experiment, 6 hours 20 minutes.

* See note on p. 407.

MANGANINE ROD.

Turned from a Bar supplied by Messrs. W. T. GLOVER. Composition ·84 Cu, ·04 Ni, ·12 Mn, approx. Density at 22° C. = 8·42.

a_2 = 1·90 centims., approx. k = ·032 to ·052, therefore b' = ·20, x_A = ·30, x_B = 2·60, x_C = 2·90, $x_{C'}$ = 3·03 centims., s = 1·83 sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat temp. difference.	$\sqrt{6 \cdot 85 \frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
°C.															°C.	
−190	0				−·0168											
−169	·180	1·037	1·84	·0426	·0807	·0972	9·95	·00428	·162	2·38	·0101	·0094	·298	·0316	−157	·0353
−165	0				−·0161											
−145	·180	1·045	1·85	·0426	·0774	·0927	9·49	·00448	·162	2·38	·0107	·0098	·298	·0329	−135	·0363
−138	0				−·0145											
−117	·180	1·050	1·85	·0426	·0746	·0877	8·98	·00474	·161	2·38	·0113	·0103	·298	·0346	−110	·0377
−110	0				−·0117											
− 94	·180	1·054	1·86	·0428	·0724	·0833	8·53	·00501	·161	2·38	·0119	·0109	·298	·0366	− 89	·0396
− 92	0				−·0101											
− 76	·179	1·060	1·86	·0428	·0709	·0798	8·17	·00524	·161	2·38	·0125	·0114	·298	·0382	− 72	·0409
− 73	0				−·0077											
− 58	·179	1·062	1·87	·0429	·0686	·0756	7·74	·00554	·161	2·38	·0132	·0121	·298	·0406	− 56	·0431
− 53	0				−·0064											
− 46	0				−·0092											
− 31	·185	1·120	2·03	·0464	·0678	·0758	7·76	·00598	·161	2·38	·0143	·0131	·298	·0440	− 30	·0460
− 28	0				−·0067											
− 14	·185	1·120	2·03	·0464	·0660	·0718	7·35	·00632	·161	2·38	·0150	·0138	·298	·0465	− 14	·0483
− 13	0				−·0050											
− 1	·185	1·126	2·05	·0469	·0654	·0690	7·16	·00655	·162	2·38	·0156	·0144	·298	·0486	− 1	·0501
− 1	0				−·0023											
+ 8	·185	1·130	2·05	·0469	·0661	·0678	6·94	·00676	·162	2·38	·0161	·0149	·298	·0502	+ 8	·0515
7	0				−·0011											
16	·184	1·130	2·04	·0465	·0652	·0658	6·73	·00691	·162	2·38	·0164	·0151	·298	·0509	16	·0520
15	0				−·0002											
22	·184	1·130	2·04	·0465	·0650	·0648	6·63	·00702	·163	2·38	·0167	·0154	·298	·0519	22	·0528
19	0				+·0007											

Duration of experiment, 7 hours.

* See note on page 407.

LIPOWITZ ALLOY ROD.

Turned from a Bar Cast from Sample of Alloy supplied by Messrs. KAHLBAUM. Approx. Composition ·50 Bi, ·25 Pb, ·14 Sn, ·11 Cd. Density at 20° C. = 9·66. Melting-point 65° C.

$a_2 = 1{\cdot}85$ centims., approx. $k = {\cdot}038$ to ${\cdot}044$, therefore $b' = {\cdot}185$, $x_A = {\cdot}29$, $x_B = 2{\cdot}51$, $x_C = 2{\cdot}80$, $x_{C'} = 2{\cdot}92$ centims.
$s = 1{\cdot}83$ sq. centims.

Mean Pt. temp. of rod.	Amp.	Volts.	Watts on sleeve.	Heat to rod, gram-degrees per sec.	$R_B - R_A$, ohms.	Differences, ohms.	Pt. temp. differences.	Heat / temp. difference.	$\sqrt{6{\cdot}85\frac{h}{k}}$.	$x_BS_B - x_AS_A$.	I.*	II.*	III.*	k_P.	Mean H. temp. of rod.	k_H.
° C.															° C.	
+ 18	0				+ ·0035											
22	·160	·967	·152	·0346	·0575	·0539	5·52	·00626	·177	2·30	·0144	·0131	·302	·0433	+ 22	·0440
19	0				·0037											
− 192	0				− ·0100											
− 175	·165	·942	·154	·0355	·0578	·0676	6·92	·00512	·150	2·28	·0117	·0109	·292	·0374	− 162	·0418
− 170	0				− ·0097											
− 156	·165	·950	·154	·0354	·0584	·0677	6·93	·00510	·153	2·28	·0116	·0108	·293	·0369	− 145	·0409
− 154	0				− ·0089											
− 141	·166	·960	·157	·0360	·0590	·0673	6·89	·00522	·155	2·29	·0120	·0112	·293	·0383	− 132	·0422
− 139	0				− ·0078											
− 126	·166	·960	·157	·0360	·0591	·0663	6·79	·00530	·157	2·29	·0122	·0113	·294	·0385	− 118	·0421
− 123	0				− ·0067											
− 112	0				− ·0080											
− 97	·166	·982	·161	·0370	·0681	·0653	6·68	·00553	·161	2·29	·0127	·0117	·296	·0395	− 92	·0428
− 94	0				− ·0065											
− 80	·166	·990	·162	·0372	·0587	·0647	8·82	·00561	·163	2·29	·0129	·0119	·297	·0400	− 76	·0429
− 76	0				− ·0056											
− 63	·166	·990	·162	·0370	·0587	·0639	6·54	·00565	·165	2·29	·0130	·0120	·297	·0404	− 60	·0428
− 60	0				− ·0047											
− 43	·166	·995	·163	·0373	·0583	·0623	6·38	·00584	·167	2·30	·0134	·0123	·298	·0412	− 41	·0434
− 41	0				− ·0036											
− 29	·165	1·000	·162	·0370	·0584	·0616	6·30	·00587	·169	2·30	·0135	·0124	·298	·0416	− 28	·0435
− 24	0				− ·0029											
− 18	0				− ·0040											
− 6	·162	·977	·156	·0357	·0526	·0563	5·76	·00619	·173	2·30	·0143	·0131	·300	·0436	− 6	·0451
− 4	0				− ·0034											
+ 7	·160	·980	·154	·0352	·0545	·0567	5·80	·00606	·175	2·30	·0140	·0128	·301	·0425	+ 7	·0436
6	0				− ·0008											
15	·161	·977	·155	·0353	·0537	·0543	5·56	·00634	·176	2·30	·0146	·0134	·301	·0445	15	·0455
14	0				− ·0004											
22	·160	·972	·153	·0348	·0534	·0537	5·50	·00632	·177	2·30	·0146	·0133	·302	·0441	22	·0449
22	0				− ·0002											

Duration of experiment, 5 hours 40 minutes.

* See note on p. 407.

The results of the observations are embodied in the curves on the opposite page.

From the smooth curves drawn as nearly as possible through the points given by the observations, the table on p. 426 of values of the conductivities at various temperatures has been constructed.

Discussion of the Thermal Conductivity Results.

An inspection of the curves of results over the temperature range −170° C. to +30° C., to which for comparison the results obtained by Jäger and Diesselhorst[*] at 18° C. and 100° C. have been added, shows that over the whole range from −170° C. to +100° C. the thermal conductivity of a pure metal changes comparatively little. As to the character and magnitude of the small rates of change of conductivity with temperature at 18° C., the two sets of experiments are in close agreement throughout. As the temperature rises there is a slight decrease in the conductivity of zinc, copper, iron, cadmium, tin, and lead over the range covered by the experiments. The absolute values of the conductivity at 18° C., given by the present experiments, agree closely with those given for these metals at this temperature by Jäger and Diesselhorst, except in the case of cadmium, for which the present value of k at 18° C. is about 2 per cent. less than that obtained by Jäger and Diesselhorst. Silver appears to have a maximum conductivity at about −90° C., and to decrease very slightly in conductivity up to +100° C. The value of k at 18° C., according to the present experiments, is about 3 per cent. less than that of Jäger and Diesselhorst. Steel shows a slight tendency to a maximum about −50° C., the change with temperature at 18° C. agreeing with that found by Jäger and Diesselhorst, although the absolute value at 18° C. is about 5 per cent. higher than for their steel. Aluminium has a minimum conductivity about −110° C., and a rate of change with temperature about 18° C., which agrees with that found by Jäger and Diesselhorst, although the absolute value is about 4 per cent. higher than theirs.

The alloys tested all increase in conductivity with increase of temperature. In the case of manganine the absolute value and the rate of increase with temperature at 18° C. agree with those given by Jäger and Diesselhorst.

When the slight unavoidable differences in purity of the specimens used by Jäger and Diesselhorst and by myself are taken into account, the close agreement between the results obtained by methods which differ so materially from each other must be considered most satisfactory, for not only have we possible differences in chemical composition to contend with, but, according to the researches of Beilby,[†] for each

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[*] W. Jäger and H. Diesselhorst, 'Wissenschaftl. Abhandl. d. Phys.-Techn. Reichsanstalt,' 3, p. 270, (1900). In cases where they give two or more values for the same metal, that value has been used to which they attach the greatest weight.

[†] 'Phil. Mag.,' 8, p. 258 (1904); 'Roy. Soc. Proc.,' A, 76, p. 462 (1906), and 79, p. 463 (1907).

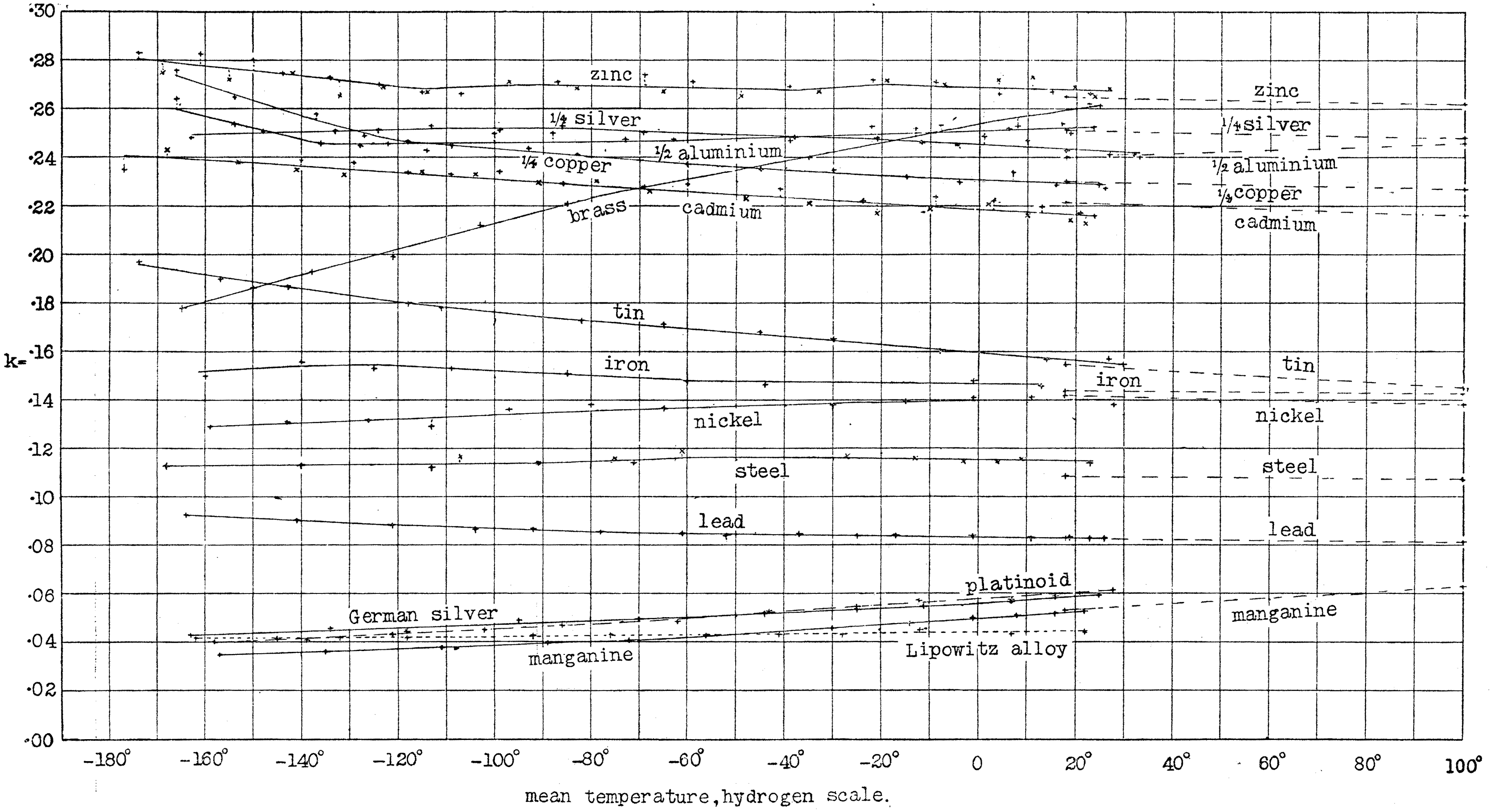

Fig. 3.

Curves of variation of thermal conductivities k of pure metals and alloys with temperature.

Observed values are indicated by crosses, thus :— +. Where a second experiment has been performed, in general with a shorter length of the rod, the observations are indicated thus :— ×.

The dotted lines to the right join the values given by Jäger and Diesselhorst for the conductivities at 18° and 100° C. respectively.

For silver and copper the values plotted are $k/4$, and for aluminium $k/2$, as indicated.

TABLE of Thermal Conductivities of pure Metals and Alloys between −170° and +18° C., as deduced from the present Experiments, compared with those at 18° and 100° C. given by JÄGER and DIESSELHORST as the results of their Experiments.

Substance.	From the present experiments.											From JÄGER and DIESSELHORST'S experiments.		
	Chemical and physical state.	At −170° C.	At −160° C.	At −150° C.	At −125° C.	At −100° C.	At −75° C.	At −50° C.	At −25° C.	At 0° C.	At +18° C.	At +18° C.	At +100° C.	Chemical and physical state.
Copper	Pure soft drawn	(1·112)	1·079	1·054	·996	·973	·958	·944	·932	·924	·916	·918	·908	Pure
Silver	·999 Ag	(·996)*	·998	1·000	1·005	1·008	1·004	·997	·997	·981	·974	1·006	·992	·9998 Ag
Zinc	Pure re-distilled cast	·280	·278	·276	·270	·271	·269	·268	·269	·269	·268	·265	·262	Pure cast
Cadmium . . .	„ „ „	·240	·239	·238	·234	·231	·227	·225	·222	·219	·217	·222	·216	„
Aluminium . .	·99 Al	(·524)	·514	·508	·491	·492	·493	·496	·499	·502	·504	·480	·492	·99 Al
Tin	Pure cast	·195	·192	·189	·182	·176	·172	·168	·164	·160	·157	·155	·145	Pure wire
Lead	Pure	(·093)	·092	·091	·089	·087	·085	·085	·084	·084	·083	·083	·082	Pure
Iron	·994 Fe, ·0015 Mn, ·0013 Ni, approx. ·001 C	(·151)	·152	·153	·154	·152	·150	·148	·147	·147	·147	·144	·143	·9955 Fe
Nickel	·99 Ni	(·128)	·129	·130	·132	·134	·136	·137	·139	·140	·140	·142	·138	·97 Ni
Steel	Approx. ·01 C	(·113)	·113	·113	·113	·114	·115	·116	·116	·116	·115	·108	·107	·01 C
Brass	Approx. ·70 Cu, ·30 Zn	(·175)	·181	·186	·200	·213	·225	·235	·244	·254	·260	—	—	—
German silver .	Approx. ·62 Cu, ·15 Ni, ·22 Zn	(·042)	·043	·044	·045	·047	·049	·051	·054	·056	·059	—	—	—
Platinoid . . .	„ „ „ „	(·039)	·040	·041	·043	·045	·048	·051	·055	·058	·060	—	—	—
Manganine . . .	Approx. ·84 Cu, ·04 Ni, ·12 Mn	(·034)	·035	·035	·037	·039	·041	·043	·046	·050	·052	·053	·063	—
Lipowitz alloy .	Approx. ·50 Bi, ·25 Pb, ·14 Sn, ·11 Cd	(·042)	·042	·042	·042	·042	·043	·043	·044	·044	·044	—	—	—

* The values enclosed in brackets are obtained by a slight extrapolation.

metal there is a hard and a soft state, with different physical properties. If in the future a still closer agreement between the results obtained by different observers is demanded, it will be necessary to specify both chemical composition and microscopic structure of the materials tested.

The fact that the thermal conductivities of most of the pure metals decrease with increase of temperature, while those of all the alloys tested increase with increase of temperature, raises the question whether the increase with temperature observed in the case of aluminium both in Jäger and Diesselhorst's experiments and in the present ones, and in that of nickel in the present experiment, cannot be attributed to small amounts (less than 1 per cent.) of impurity remaining in the specimens tested.

While there is much to be said in favour of this in the two cases mentioned, it seems to me that the apparent existence of a maximum of conductivity of silver, and possibly of steel, and a minimum conductivity of aluminium, shows that we must guard ourselves against stating that the thermal conductivities of all pure metals decrease as the temperature rises, and limit ourselves to the less general statement that the thermal conductivities of most pure metals decrease with rise of temperature over the range −170° C. to +100° C., while those of all the alloys tested increase over the same range of temperature.

On comparing these results with those obtained previously for non-metallic substances,* mostly electrical insulators, it will be seen that on the whole metals and non-metals are affected in the same way by change of temperature, *i.e.*, both tend to conduct heat better at low than at high temperatures, while alloys conduct better at high than at low temperatures, and in this respect resemble glass, the only mixture of non-conductors of electricity which has up to the present been investigated.

Section 2.—The Electrical Conductivity Measurements.

The effect of temperature on the electrical resistivity of the material of each rod was determined by sending an electric current along the rod and through a standard small resistance in series with it. The rod was supported on a pair of knife edges, 4 centims. apart, and the difference of potential between the two was compared with that between the potential terminals of the standard resistance by the method shown in fig. 4.

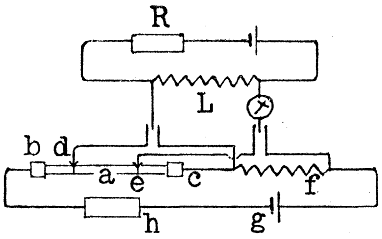

Fig. 4.

a is the rod, *b*, *c* two short copper cylinders provided with slightly conical holes into which the ends of the rod were gently forced, *d*, *e* are the knife edges which support the rod and serve as potential terminals, *f* is the standard resistance of 0·000994 or 0·0001182 ohm. The current was supplied from a storage cell *g*, and could be adjusted by means of the resistance *h*.

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* Lees, 'Phil. Trans. Roy. Soc.,' A, 191, p. 399 (1898); A, 204, p. 433 (1905).

A second circuit, insulated from the first, consisted of a small resistance L provided with potential terminals, a storage cell, and a resistance box R.

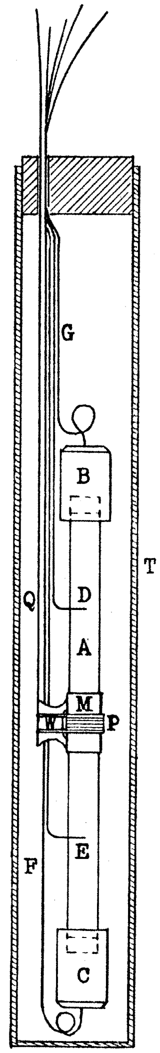

Fig. 5.

One of the potential terminals of L could be connected directly to one of the potential terminals of *f*, or to one of the knife edges on the rod, while the other could be connected through a low-resistance galvanometer to the other terminal of *f*, or to the other knife edge.

In either case the resistance R was adjusted till no current passed through the galvanometer. If R_R is the resistance which gives a balance on the rod and R_S that on the standard resistance, the resistance of the rod between the knife edges is equal to R_S/R_R times that of the standard resistance, the electromotive force in each circuit being supposed constant during the observations, and the resistances of the cell and L being negligible compared to R.

In order to overcome the effects of thermo-electric currents the circuits were made through a "thermo-electric key," which, when up, cuts the cells out of but leaves the galvanometer in circuit. On depressing it, the first operation is to break the galvanometer circuit, then make both cell circuits, and lastly re-make the galvanometer circuit.

The frame supporting the rods during the test is shown in fig. 5. It consists of four strips, G, D, E, F, of sheet copper, 1 centim. wide, 0·08 and 0·05 centim. thick, insulated from each other by strips of mica slightly wider than the copper, the whole shellacked and bound together with shellacked silk thread.

The upper end of each strip is soldered to 30 centims. of copper wire of ·054 centim. diameter, which connects it with the rest of the circuit shown in fig. 4.

The outer copper strips, F and G, end in short lengths of flexible conductor, to the ends of which the copper caps B, C, which fit on to the rod A, are soldered. The middle copper strips are bent at right angles at their lower ends and form the two knife edges D and E in which the rod is supported. To insure good contact, the rod is bound to the frame by a few turns of silk thread passing over it at each knife edge.

The temperature of the rod was measured at the centre of its length by a platinum thermometer P, consisting of about 31 centims. of No. 40 silk-covered platinum wire identical with that used in the thermal conductivity observations, and having a resistance at 0° C. of about 2·73 ohms. This was wound at the centre of a thin ring of mica which just fitted the rods. It was made by winding round one of the rods, whose surface was oiled slightly, a very thin strip of mica which had been shellac varnished, binding wire round it and baking it. On cooling, a ring which stood throughout the experiments was obtained. The ends of the fine platinum wire were soldered to the ends of two lengths each of 33 centims. of double silk-covered copper wire, 0·069 centim. in diameter, Q. In order to prevent strain of the thin wire, the copper wires rested in grooves in the edge of a small block of wood, W, the other edge of which rested on the mica ring, and several turns of thread were taken round ring, block, and leads. The whole was shellac varnished and stoved.

The platinum thermometer formed one arm of a resistance bridge, the equal arms of which each consisted of an ohm coil. The adjustable arm consisted of two resistance boxes with units and tenths, and the dial mercury cup resistance (p. 384) giving hundredths. The current was supplied by a Leclanché cell through a resistance of 5 to 10 ohms, and the galvanometer was a small pointer galvanometer allowing a determination of resistance to 0·002 ohm, and of temperature, therefore, to 0·2° C.

The frame, rod, and thermometer were enclosed in a brass tube T, 15·7 centims. long, 2·2 centims. outside diameter, with walls about 0·05 centim. thick, lined inside with mica. Around this tube a few turns of insulated manganine wire were wound. Through this wire an electric current could be sent if at any time it was necessary to raise the temperature of the apparatus. The rod was not concentric with the tube, owing to the space occupied by the frame. The space between rod and tube was filled with clean sand so as to facilitate communication of heat between the two. The upper end of the tube, out of which the insulated leads passed, was closed by a split cork.

Method of Experimenting.

In making an experiment the following was the routine adopted :—

The rod was first lightly oiled at the centre of its length, and the mica ring of the platinum thermometer slipped over it, the oil assisting the motion and excluding air from the space between rod and inner surface of ring. The excess of oil was then wiped away. The knife edges were then cleaned if necessary. The ends of the rod were next inserted into the end caps and the rod then placed on the knife edges. The threads holding the rod in its place were then wound round it, the rod moved slightly to ensure good electrical contact between it and the knife edges, and the rod and frame placed in the brass tube. The sand was then poured in and the top of the tube closed. The brass tube and contents were then placed in a straight Dewar vacuum vessel, 20 centims. deep, 4 centims. wide, and the top closed by a plug of cotton wool. The various leads were brought to copper mercury cups at which connections were made to the circuits.

The temperature of the rod was determined by balancing the bridge, and the resistance in series with the rod was so adjusted that the values of R (fig. 4) necessary to balance when the supplementary resistance L was connected to the rod and standard resistance *f* respectively were convenient. After a few minutes' interval, during which any temperature differences in parts of the apparatus had time to disappear, the temperature and the values of R for balance were determined accurately.

Liquid air was then poured into the Dewar tube, and when the temperature had become steady, the observations of temperature and of values of R were repeated.

The liquid air was then poured out of the Dewar tube, and the temperature rose slowly. During the rise several observations of temperature and of R were made at intervals. After three or four observations had been taken the temperature of the apparatus was raised 40° or 50° by sending a current for a few minutes through the manganine wire wound on the outside of the brass tube. When the rate of rise of temperature was again normal, observations were again taken. These steps were repeated till the original temperature of the rod was reached. In no case did the value of the resistivity at the end differ appreciably from its value at the beginning of an experiment.

Reduction of the Electrical Conductivity Observations.

In order to obtain the temperature of the rod from the measurements of the resistance of the platinum thermometer and its leads, we require to know the resistance of the leads and the constants of the thermometer. The resistance of the leads was determined by placing an equal length of the same wire alongside the leads and determining its resistance for various temperatures of the rod during one of the experiments. The values thus found were subtracted from the resistances determined by the bridge in each experiment.

The constants of the platinum coil were found by comparison with the two coils previously used in the heat conductivity measurements. The three coils were made in the same way of the same wire, and the new coil agreed throughout with the old "B" coil. The constants used in the reduction of the resistance measurements to temperature in the former experiments were therefore used in the present case.

As it was not the practice to wait till the temperature was quite steady before observations of temperature and resistance were made, there may be a slight difference of temperature between the rod and thermometer. Attempts were made to determine the magnitude of this difference by raising or lowering the temperature of the zinc rod at about the same rate as was usual in an experiment, *i.e.*, 1° C. per minute, and comparing the results. No difference of temperature of the thermometer so large as the experimental error, ·2° C., could be detected between observations which gave the same value of the resistance of the rod with rising or falling temperature. A rough calculation from the dimensions of the mica ring on which the thermometer was wound showed that the difference of temperature of rod and thermometer could not exceed one-tenth of the change of temperature of the rod in one minute. Whenever in an experiment the rise of temperature has exceeded ·5° C.

per minute, a correction equal to one-tenth the rate of rise has been subtracted from the observed temperature.

The determination of the resistance of the rods depends virtually on the measurement of the current through them and the difference of potential at two points 4 centims. apart. As the rods are about 7·5 centims. long, and the current was introduced through a strip about 1 millim. broad, situated at the ends of the curved surfaces of the rods, the current may, to the degree of accuracy aimed at in the present paper, be assumed to be uniformly distributed throughout the cross section of the rod in the interval between the two knife edges at which the potential difference is measured.[*] If R_R is the resistance found, ρ the resistivity of the material, A the area of cross section of the rod, and $2a$ the distance between the knife edges, we may therefore write $\rho = R_R A/2a$.[†]

---

[*] The distribution of potential v throughout a rod of length $2l$ and radius R and resistivity ρ, through which an electric current flows which enters and leaves by strips of breadth b of the curved surface at each end, and is of uniform density c at each strip, is given by the equation

$$v = \frac{8cl\rho}{\pi^2}\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)^2}\sin(2n+1)\frac{\pi b}{2l}\sin(2n+1)\frac{\pi x}{2l}\,I_0\left((2n+1)\frac{\pi r}{2l}\right)\Big/ I_1\left((2n+1)\frac{\pi R}{2l}\right),$$

where x is the distance of a point from the central transverse section, r is its distance from the axis of the rod, and $I(r)$ is the Bessel function with unreal argument.

Hence the potentials at the points $\pm a$, R on the surface at which the knife edges touch the rod differ from that at the central section by

$$v_a = \frac{8cl\rho}{\pi^2}\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)^2}\sin(2n+1)\frac{\pi b}{2l}\sin(2n+1)\frac{\pi a}{2l}\,I_0\left((2n+1)\frac{\pi R}{2l}\right)\Big/ I_1\left((2n+1)\frac{\pi R}{2l}\right),$$

or, if C is the total current $(=2\pi Rbc)$,

$$v_a = \frac{4Cl\rho}{\pi^3 bR}\sum_{n=0}(-1)^n\frac{1}{(2n+1)^2}\sin(2n+1)\frac{\pi b}{2l}\sin(2n+1)\frac{\pi a}{2l}\,I_0\left((2n+1)\frac{\pi R}{2l}\right)\Big/ I_1\left((2n+1)\frac{\pi R}{2l}\right).$$

In the apparatus used $l = 3\cdot75$, $a = 2$, $b = \cdot1$, $R = \cdot29$ centim.

Hence

$$\frac{\pi b}{2l} = 2^\circ\cdot4, \quad \frac{\pi a}{2l} = 48^\circ, \quad \frac{\pi R}{2l} = \cdot121,$$

and the equation reduces to

$$v_a = \frac{C\rho a}{\pi R^2} \text{ to within 1 part in 500.}$$

The above statement is therefore correct to 1 part in 500.

[†] If R is the radius of the rod, and $2a$ the distance apart of the knife edges at 0° C., at any temperature θ° C. we have

$$\rho = R_R A/2a\,[1+(2\alpha-\beta)\,\theta],$$

where α is the mean coefficient of expansion of the material of the rod, and β that of the copper frame holding it, between 0° C. and θ° C.

The values of the mean coefficients of expansion of metals at low temperatures are only known in a few cases, but from these cases it does not seem likely that the value of the correcting factor $1+(2\alpha-\beta)\,\theta$ exceeds 1 by more than 1 part in 200, in any of the experiments.

Results of Electrical Conductivity Experiments.

The following tables of resistivities at different temperatures embody the results of the observations :—

Silver Rod.

Ag = ·999.

Density at 21° C. = 10·47.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
– 21·3	$1·684 \times 10^{-6}$	– 109·0	$0·880 \times 10^{-6}$
– 176·1	0·456	– 102·5	·923
– 178·1	·455	– 98·6	·942
– 178·5	·453	– 54·9	1·236
– 178·3	·472	– 52·8	1·239
– 177·9	·471	– 50·6	1·245
– 177·6	·470	– 13·0	1·468
– 177·5	·470	– 12·0	1·471
– 151·2	·609	+ 21·0	1·675
– 144·9	·660	21·0	1·676
– 139·2	·693		

Aluminium Rod.

Al = ·99.

Density at 20° C. = 2·70.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 21·1	$2·97 \times 10^{-6}$	– 113·3	$1·356 \times 10^{-6}$
20·7	2·97	– 108·9	1·415
– 181·5	0·623	– 103·0	1·480
– 178·3	·645	– 40·5	2·256
– 169·3	·711	– 36·0	2·301
– 162·2	·782	– 35·0	2·301
– 153·9	·885	– 33·9	2·305
– 146·6	·978	+ 12·2	2·86

Copper Rod.

Soft-drawn High-conductivity Copper.

Density at 23° C. = 8·84.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
– 176·8	$0·375 \times 10^{-6}$	– 133·4	$0·677 \times 10^{-6}$
– 175·7	·375	– 130·3	·707
– 173·9	·384	– 105·0	·884
– 173·0	·394	– 102·0	·909
– 172·7	·398	– 97·0	·936
– 171·1	·401	– 91·1	·989
– 159·0	·484	– 36·0	1·365
– 155·2	·508	– 32·7	1·388
– 151·2	·543	– 17·0	1·502
– 143·9	·579	– 15·5	1·506
– 141·6	·612	+ 16·9	1·753
– 136·9	·637	16·9	1·750

Zinc Rod.

Pure Redistilled.

Density at 21° C. = 7·10.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 18·8	$6·30 \times 10^{-6}$	– 66·9	$4·41 \times 10^{-6}$
– 180·3	1·699	– 62·9	4·46
– 179·2	1·732	– 24·7	5·36
– 174·6	1·833	– 20·3	5·48
– 171·5	1·931	– 17·6	5·52
– 168·4	1·96	+ 16·7	6·30
– 168·2	1·96	18·8	6·34
– 116·3	3·26	47·8	6·99
– 107·0	3·50	51·7	7·09
– 99·7	3·69	54·3	7·14
– 87·2	3·93	90·3	8·01
– 78·6	4·14	90·0	8·03
– 70·1	4·32	88·8	7·99

CADMIUM ROD.

Pure Redistilled.

Density at 21° C. = 8·64.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 21·0	$7{\cdot}72 \times 10^{-6}$	− 75·1	$5{\cdot}05 \times 10^{-6}$
21·0	7·72	− 70·1	5·15
− 178·1	2·22	− 65·4	5·28
− 174·1	2·34	− 59·9	5·46
− 171·0	2·41	− 32·5	6·18
− 167·9	2·48	− 29·3	6·28
− 165·9	2·56	− 25·2	6·38
− 125·4	3·67	− 23·5	6·44
− 116·9	3·87	− 5·7	6·96
− 110·5	4·07	+ 22·8	7·78
− 105·8	4·18	22·8	7·78

TIN ROD.

Pure Tin (KAHLBAUM).

Density at 21° C. = 7·28.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 11·6	$10{\cdot}65 \times 10^{-6}$	− 98·6	$5{\cdot}96 \times 10^{-6}$
− 170·4	3·00	− 91·3	6·24
− 170·4	3·00	− 87·7	6·44
− 169·1	3·17	− 52·9	7·88
− 165·9	3·26	− 48·8	8·08
− 164·7	3·36	− 46·7	8·13
− 162·3	3·42	− 42·2	8·29
− 163·1	3·45	− 40·0	8·42
− 150·8	3·51	− 36·1	8·59
− 148·8	3·96	+ 11·8	10·70
− 107·0	5·58	13·3	10·75
− 103·1	5·77	13·3	10·75

BRASS ROD.

Approx. ·70 Cu, ·30 Zn.

Density at 22° C. = 8·44.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 21·5	$6{\cdot}57 \times 10^{-6}$	− 92·3	$5{\cdot}06 \times 10^{-6}$
21·4	6·57	− 87·2	5·13
− 174·5	3·84	− 78·6	5·20
− 171·0	3·89	− 76·6	5·23
− 165·1	3·97	− 68·8	5·34
− 161·2	4·03	− 29·3	5·89
− 158·9	4·09	− 25·7	5·93
− 150·8	4·16	− 22·7	5·98
− 109·1	4·78	− 20·4	6·02
− 104·6	4·84	+ 14·9	6·50
− 99·5	4·94		

IRON ROD.

Fe = ·994.

Density at 21° C. = 7·74.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 19·9	$14{\cdot}33 \times 10^{-6}$	− 81·8	$8{\cdot}98 \times 10^{-6}$
− 175·3	4·90	− 77·6	9·15
− 175·8	4·87	− 71·4	9·48
− 175·5	4·86	− 69·5	9·56
− 175·6	4·86	− 66·9	9·67
− 174·2	4·89	− 61·4	9·96
− 170·3	5·01	− 21·8	12·03
− 166·3	5·20	− 19·2	12·19
− 159·4	5·45	− 16·1	12·33
− 154·4	5·64	− 14·0	12·46
− 129·6	6·76	+ 10·3	13·70
− 121·4	7·08	22·1	14·46
− 117·0	7·34	21·0	14·30
− 111·1	7·59	18·3	14·21
− 106·7	7·81	16·7	14·16
− 99·7	8·16		

NICKEL ROD.

Ni = ·99.

Density at 21° C. = 8·80.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 17·7	12·87 × 10^{-6}	− 70·3	9·04 × 10^{-6}
− 178·1	5·42	− 65·0	9·29
− 178·7	5·41	− 60·6	9·48
− 181·7	5·29	− 50·0	9·92
− 158·6	5·92	− 41·3	10·26
− 147·3	6·24	− 39·8	10·36
− 137·1	6·61	− 34·5	10·61
− 117·5	7·22	− 28·3	10·86
− 107·8	7·76	− 24·1	11·02
− 101·8	7·94	+ 19·3	12·92
− 94·3	8·24	22·1	13·05
− 88·4	8·42	22·1	13·07
− 79·8	8·74		

LEAD ROD.

Pure.

Density at 25° C. = 11·29.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 17·4	20·9 × 10^{-6}	− 92·2	12·7 + 10^{-6}
17·2	20·9	− 89·2	12·9
− 167·2	6·98	− 55·4	15·3
− 170·0	6·71	− 53·7	15·5
− 137·7	9·01	− 53·0	15·6
− 135·7	9·25	− 51·8	15·7
− 132·7	9·45	− 14·5	18·5
− 129·4	9·71	− 14·0	18·5
− 96·0	12·3	+ 15·7	20·8
− 94·6	12·6		

STEEL ROD.

C approx. ·01.

Density at 24° C. = 7·84.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 16·2	18·33 × 10^{-6}	− 101·9	11·13 × 10^{-6}
− 172·7	7·19	− 95·8	11·49
− 173·9	7·12	− 92·7	11·69
− 172·2	7·18	− 40·8	14·74
− 165·2	7·54	− 39·6	14·84
− 154·8	8·07	− 36·0	15·08
− 146·9	8·36	+ 21·8	18·67
− 111·0	10·53	22·1	18·71

PLATINOID ROD.

Approx. ·62 Cu, ·15 Ni, ·22 Zn.

Density at 22° C. = 8·66.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 21·2	34·4 × 10^{-6}	− 99·5	33·1 × 10^{-6}
− 171·0	32·4	− 96·4	33·1
− 173·0	32·3	− 92·1	33·3
− 172·0	32·4	− 71·0	33·3
− 160·2	32·3	− 66·6	33·5
− 155·0	32·4	− 35·0	33·7
− 146·7	32·6	− 24·4	33·8
− 134·6	32·8	+ 10·0	34·3
− 103·9	33·0	10·5	34·3

German Silver Rod.

Approx. ·62 Cu., ·15 Ni, ·22 Zn.

Density at 22° C. = 8·67.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 17·2	$39 \cdot 9 \times 10^{-6}$	− 116·1	$38 \cdot 3 \times 10^{-6}$
17·2	39·8	− 105·3	38·4
− 169·1	38·0	− 95·5	38·6
− 168·2	38·0	− 57·2	39·0
− 167·7	38·0	− 53·3	39·1
− 159·3	38·0	− 50·9	39·1
− 153·0	37·9	− 16·5	39·5
− 145·8	37·9	− 15·5	39·5
− 134·6	38·0	+ 16·7	39·8
− 126·9	38·1	16·3	39·8

Manganine Rod.

Approx. ·84 Cu, ·04 Ni, ·12 Mn.

Density at 22° C. = 8·42.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 17·7	$44 \cdot 6 \times 10^{-6}$	− 66·8	$44 \cdot 4 \times 10^{-6}$
17·7	44·6	− 51·8	44·5
− 161·0	43·3	− 50·2	44·5
− 164·0	43·0	− 46·3	44·5
− 163·5	43·0	− 43·6	44·5
− 163·0	43·0	− 26·0	44·5
− 154·5	43·1	− 17·5	44·5
− 145·3	43·2	− 14·0	44·5
− 133·6	43·4	− 13·5	44·5
− 119·2	43·9	+ 13·6	44·5
− 109·9	44·0	13·8	44·5
− 98·5	44·2		

Lipowitz Alloy Rod.

Approx. ·50 Bi, ·25 Pb, ·14 Sn, ·11 Cd.

Density at 20° C. = 9·66.

Temp., H. scale.	Resistivity, ohm per centim. cube.	Temp., H. scale.	Resistivity, ohm per centim. cube.
° C.		° C.	
+ 19·6	$47 \cdot 5 \times 10^{-6}$	− 122·9	$28 \cdot 9 \times 10^{-6}$
− 176·6	21·3	− 92·1	33·2
− 178·1	21·1	− 89·1	33·7
− 178·5	21·0	− 87·0	34·0
− 178·5	20·9	− 85·1	34·3
− 175·7	21·0	− 37·4	40·4
− 173·4	21·5	− 35·7	40·9
− 170·5	21·8	− 35·0	41·0
− 141·5	26·1	− 34·6	41·0
− 134·6	27·4	+ 12·1	46·7
− 129·3	27·9	12·1	46·8
− 125·4	28·3		

The results given in the foregoing tables are shown graphically in Plate 30, in which the crosses indicate values obtained by observation. To get a more compact diagram the scale has been varied to suit the substances tested, and the words which follow the names of the substances on the diagram, *e.g.*, "1/10 scale," indicate that the values plotted in the diagram are that particular fraction of *e.g.*, 1/10 of the values observed.

The values found by JÄGER and DIESSELHORST at 18° C. and 100° C. are shown by the broken lines to the right. The lines for the same material in the two cases are joined by a dotted line or by a bracket.

As the electrical resistivity of a metal is nearly proportional to the absolute temperature, *i.e.*, the electrical conductivity multiplied by the absolute temperature nearly a constant, and as this product is required in the discussion which follows, a table of values of the quantity at a number of temperatures within the range covered by the experiments is given on p. 437. It has been calculated from the values of the resistances taken from the curves of Plate 30.

Discussion of the Electrical Results.

A comparison of the values of the electrical resistivities obtained in the present experiments with those obtained at 18° C. and 100° C. by JÄGER and DIESSELHORST, shows that the agreement is not quite so good as in the case of the thermal measurements. This is not likely to be due to errors of observation, as the electrical measurements admit of greater accuracy than the thermal. It seems rather to point to the greater influence on the electrical properties than on the thermal, of small differences in chemical composition, or in physical structure. This is in keeping with the results obtained by GRÜNEISEN* in the cases of copper and iron.

The influence of impurity on the electrical resistivities of metals is, in fact, so marked, particularly at low temperatures,† that a comparison of the resistivities found in the present experiments with those found by DEWAR and FLEMING (D.F.), and by KAMERLINGH ONNES and CLAY (O.C.), will serve as a test of the degree of purity of the materials used.

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[*] GRÜNEISEN, 'Ann. der Phys.,' 3, p. 43 (1900).

[†] MATTHIESSEN, 'Ann. der Phys.,' 122, pp. 19, 68 (1864). DEWAR and FLEMING, 'Phil. Mag.,' 36, p. 271, (1893); FLEMING, 'Proc. Roy. Inst.,' June, 1896, p. 9. KAMERLINGH ONNES and CLAY, 'Comm. Leiden,' No. 99^c (1907).

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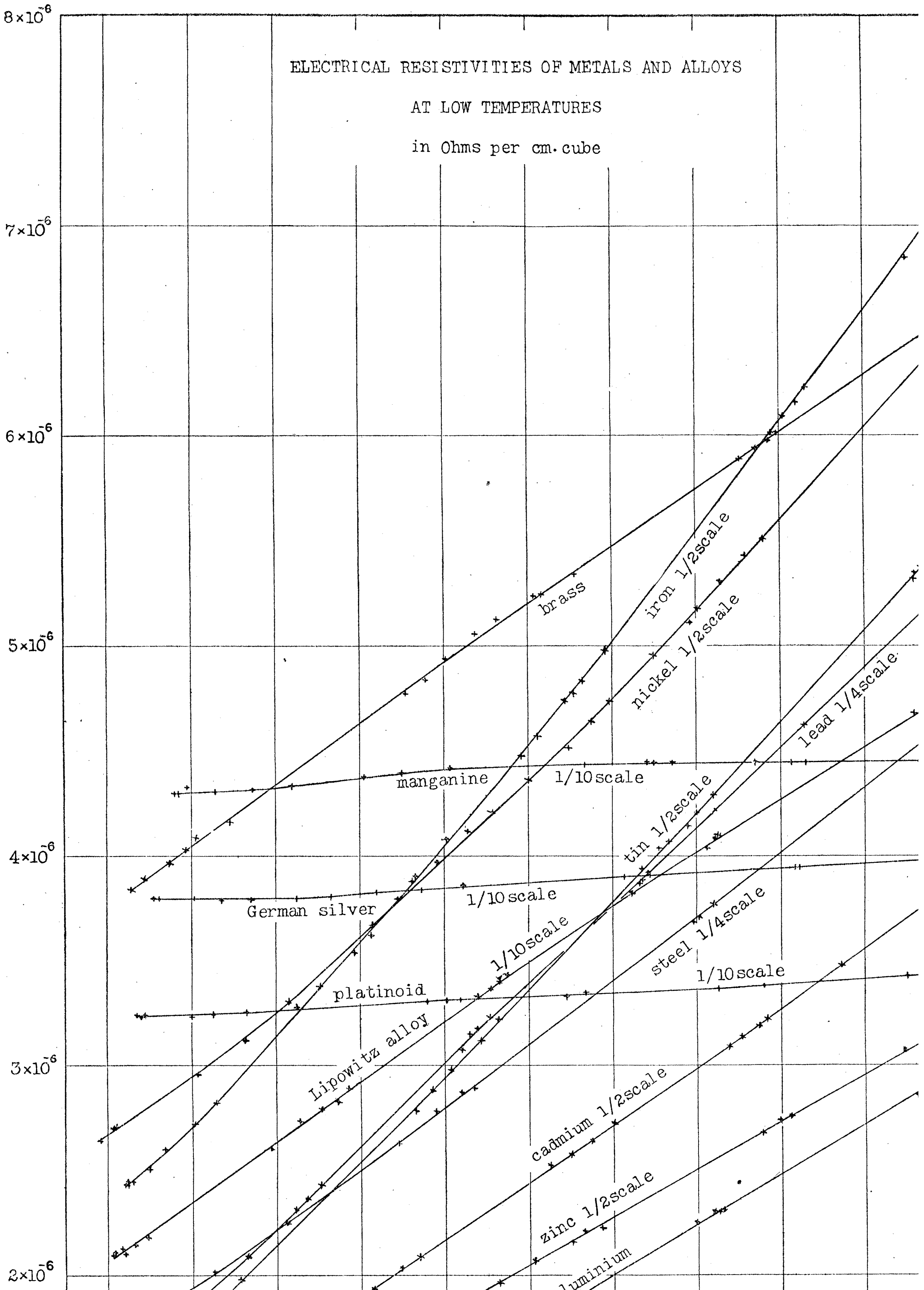

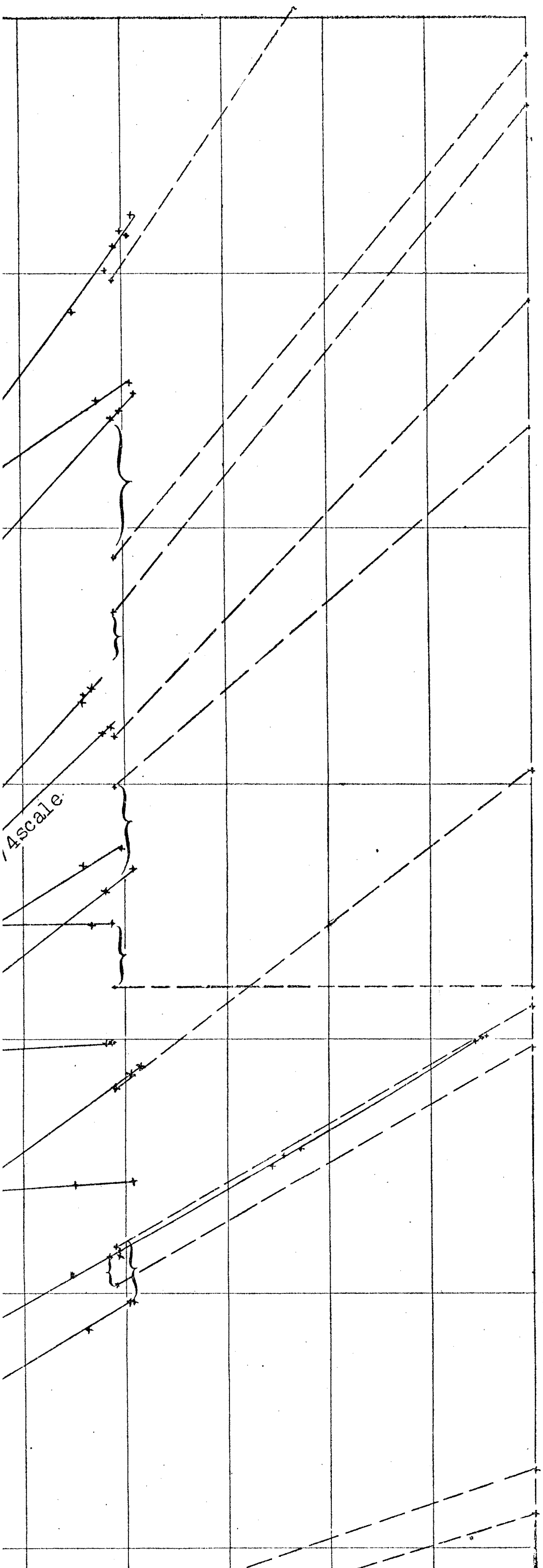
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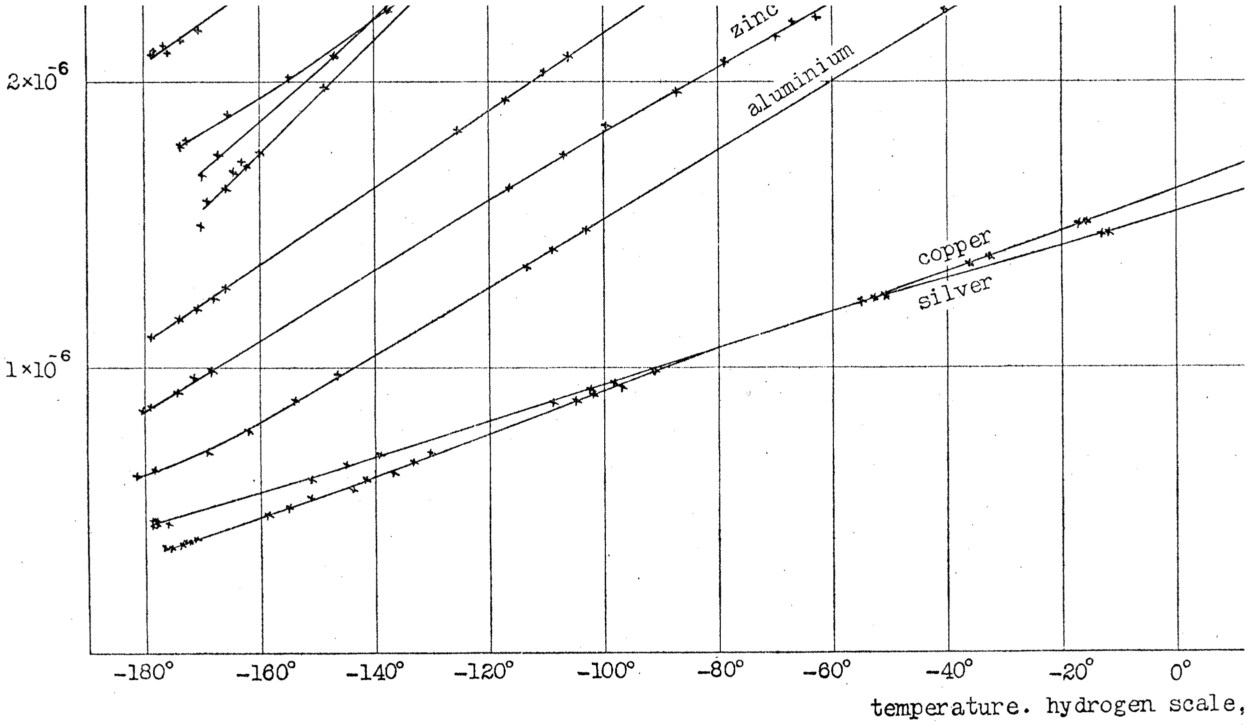

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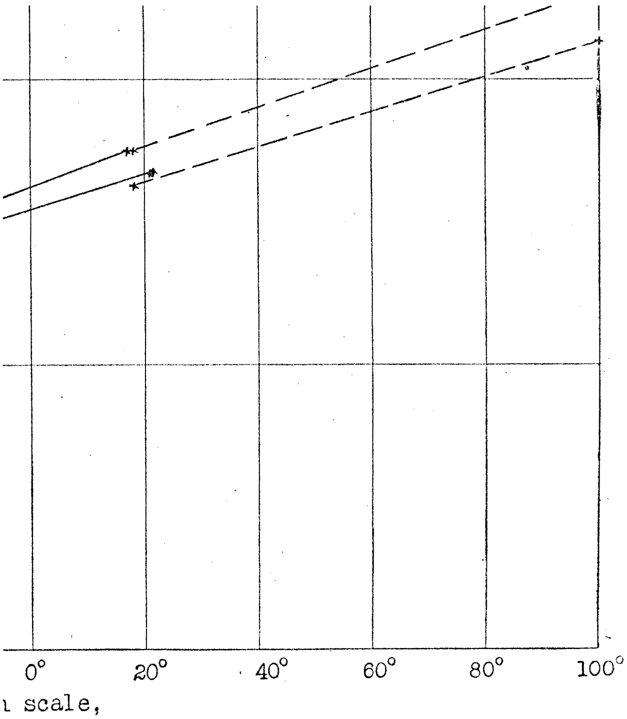
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scale,

TABLE of Values of Electrical Conductivity Multiplied by Absolute Temperature for Metals and Alloys.

Conductivities in Reciprocal Ohms per Centimetre Cube.

Temperatures on Hydrogen Scale.

Temperatures . . .	− 170° C.	− 160° C.	− 150° C.	− 125° C.	− 100° C.	− 75° C.	− 50° C.	− 25° C.	0° C.	+ 18° C.
Copper	251	230	224	202	188	180	175	172	169	165 $\times 10^6$
Silver	204	200	197	188	184	180	177	178	176	175
Zinc	53·2	51·5	50·5	48·5	47·6	46·9	46·7	46·3	46·1	46·1
Cadmium	42·2	41·5	41·2	40·5	39·8	39·4	39·1	38·8	38·3	38·0
Aluminium . . .	147	139	132	121	114	109	105	102	100	99
Tin	33·0	32·3	31·3	30·3	29·3	28·7	27·9	27·4	26·9	26·6
Lead	15·3	15·2	15·1	14·7	14·3	14·2	14·1	14·0	13·9	13·9
Iron	20·4	20·9	21·1	21·3	21·4	21·3	21·1	20·9	20·7	20·6
Nickel	18·3	19·2	20·0	21·1	21·7	22·2	22·5	22·6	22·6	22·6
Steel	14·1	14·5	14·8	15·2	15·5	15·6	15·7	15·7	15·8	15·8
Brass	26·4	27·9	29·4	32·5	35·1	37·6	39·8	41·7	43·5	44·4
German silver . .	2·70	2·98	3·24	3·88	4·50	5·10	5·71	6·29	6·90	7·30
Platinoid	3·17	3·48	3·77	4·50	5·24	5·95	6·67	7·35	8·00	8·47
Manganine . . .	2·39	2·62	2·85	3·39	3·92	4·46	5·03	5·59	6·13	6·54
Lipowitz alloy . .	4·65	4·81	4·93	5·18	5·38	5·56	5·75	5·92	6·02	6·13

TABLE of Electrical Resistivities, and of the Ratio of Resistivities at –170° C. and at 0° C.

ρ in ohms per centimetre cube.

	ρ_0.	ρ_{-170}/ρ_0.		ρ_0.	ρ_{-170}/ρ_0.
Copper . . .	$1 \cdot 62 \times 10^{-6}$	·253	Tin	$10 \cdot 16 \times 10^{-6}$	·307
	1·56	·241 D.F.		13·1	·310 D.F.
	1·58	N.*			
			Lead	19·6	·345
Silver	1·55	·326		20·4	·343 D.F.
	1·49	·317 D.F.		—	·345 O.C.
	—	·305 O.C.		19·8	N.
	1·51	N.			
			Iron	13·16	·383
Aluminium . .	2·72	·257		9·11	·188 D.F.
	2·58	·272 D.F.		10·7	N.
	2·62	N.			
			Nickel . . .	12·08	·465
Zinc	5·92	·328		12·35	·207 D.F.
	5·77	·312 D.F.		12·0	N.
Cadmium . .	7·12	·343			
	10·0	·343 D.F.			

The values of the ratio ρ_{-170}/ρ_0 in the case of DEWAR and FLEMING's metals have been calculated from their tables of results with the help of CALLENDAR's table,[†] connecting their platinum temperatures with hydrogen temperatures.

From this table it follows on the strength of MATTHIESSEN's rule that impurity raises the resistivity, that the materials used in the present experiments were, in some cases less and in some cases more pure than those used by DEWAR and FLEMING. In most cases the difference is not large, but in the cases of cadmium and tin the materials used have a considerably less resistivity, and in the case of iron a considerably greater resistivity than those used by DEWAR and FLEMING. In the case of nickel and iron the ratios of the resistivity at –170° C. to that at 0° C., are much larger than the corresponding ratios calculated from their observations. The total amount of impurity present in each of these specimens is less than 1 per cent., so that the influence of even small quantities of impurity seems to be very marked.

The table of values of the product of electrical conductivity into absolute temperature, given on p. 437, shows a general tendency for the product to increase with decrease of temperature for all the metals tested except iron and nickel, and these contained a little impurity. The increase is relatively small in the case of lead,

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[*] NICCOLAI's results, 'Accad. Lincei Atti,' 16, p. 906 (1907). His results at low temperatures have not been used, as there is no indication that the temperature readings of the thermo-junctions used by him have been reduced to the hydrogen scale.

[†] CALLENDAR, 'Phil. Mag.,' 47, p. 214 (1899).

which, however, was very free from impurity. All the alloys show a decrease of the product as the temperature decreases.

A comparison of this table with the table of thermal conductivities given on p. 426 will show that there is some similarity between the two.

SECTION III.—COMPARISON OF THERMAL AND ELECTRICAL CONDUCTIVITIES.

In 1882 L. LORENZ[*] showed by measuring both the thermal and electrical conductivities of bars of a number of different metals, that the quotient of the thermal by the electrical conductivity was not constant, as WIEDEMANN and FRANZ[†] had supposed, but increased with the temperature, so as to be nearly proportional to the absolute temperature, a result which he had anticipated[‡] on theoretical grounds. His results did not attract much attention till the publication of RIECKE's electron theory of conduction in 1898[§] and the confirmation of LORENZ's theory for pure metals between the temperatures 18° C. and 100° C. by JÄGER and DIESSELHORST, in 1899,[‖] by a method which gives the quotient of the two conductivities directly.

Next year DRUDE's electron theory appeared,[¶] and more recently H. A. LORENTZ has published one.[**]

According to these theories the electricity and energy are supposed to be carried entirely by free electrons which move to and fro and come into collision with the molecules of the metal and with each other, and are assumed to have on the average the same kinetic energy of translation as the molecules of a gas at the same temperature. In the first two theories both positive and negative electrons are movable, in the third the negative only.

On the former theories if we limit ourselves to two kinds of electrons carrying charges e_1 and e_2 where $e_2 = -e_1 = -e$, and if n_1, n_2 are the numbers per cubic centimetre, u_1, u_2 their velocities at a temperature t, l_1, l_2 their mean free paths, we have the thermal conductivity

$$k = \tfrac{1}{3}\alpha\,(n_1u_1l_1+n_2u_2l_2),$$

and the electrical conductivity

$$\kappa = (\alpha t)^{-1}\,(n_1u_1l_1e_1^{\,2}+n_2u_2l_2e_2^{\,2}),$$

where α is the mean kinetic energy of translation of a gas molecule at absolute temperature 1°.

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[*] L. LORENZ, 'Ann. der Phys.,' 13, p. 422 (1882).

[†] G. WIEDEMANN and R. FRANZ, 'Ann. der Phys.,' 89, p. 497 (1853).

[‡] L. LORENZ, 'Ann. der Phys.,' 147, p. 429 (1872).

[§] E. RIECKE, 'Ann. der Phys.,' 66, p. 353 and 545 (1898).

[‖] W. JÄGER and H. DIESSELHORST, 'Sitz.-Ber. Akad. Wiss. Berlin,' 38, p. 719 (1899); and 'Abhand. der Phys.-Tech. Reichsanstalt,' 3, p. 282 (1900).

[¶] P. DRUDE, 'Ann. der Phys.,' 1, p. 566, 3, p. 369 (1900).

[**] H. A. LORENTZ, 'Proc. Amsterdam,' 7, p. 438, &c. (1905).

Hence

$$k/\kappa t = \tfrac{4}{3}\,(\alpha/e)^2.$$

The value of e, the charge on an electron, is known with a fair degree of accuracy to be $1{\cdot}1\times10^{-20}$ electromagnetic units,* but the value of α is still uncertain, as it depends on the number n of molecules in a cubic centimetre of a gas ($\alpha = 3\,.\,\text{pressure}/2n\,\text{abs. temp.}$). The following table gives a few of the values which have been found for n by various methods and the values of α and of $k/\kappa t$ calculated from them :—

Author.	Method.	n.	α.	$k/\kappa t$.
KELVIN†	Viscosity of gas and density when condensed	1×10^{20}	$5{\cdot}6\times10^{-17}$	$3{\cdot}3\times10^{-9}$
KELVIN‡	Effect of atmosphere on light . .	$>2{\cdot}4\times10^{19}$	$<2{\cdot}3\times10^{-16}$	$<5{\cdot}9\times10^{-8}$
JEANS§	Viscosity of gas and VAN DER WAALS' b	$4{\cdot}6\times10^{19}$	$1{\cdot}2\times10^{-16}$	$1{\cdot}6\times10^{-8}$
PLANCK‖	Radiation theory and entropy . .	$2{\cdot}8\times10^{19}$	$2{\cdot}0\times10^{-16}$	$4{\cdot}3\times10^{-8}$
LORENTZ¶ . . .	Radiation theory	—	$1{\cdot}2\times10^{-16}$	$1{\cdot}6\times10^{-8}$
J. J. THOMSON** H. A. WILSON††	Determination of e and assumption that atom of H in electrolysis carries charge e .	4×10^{19}	$1{\cdot}4\times10^{-16}$	$2{\cdot}2\times10^{-8}$
	‡‡ Ditto, but assuming charge on H atom to be $2e$	2×10^{19}	$2{\cdot}8\times10^{-16}$	$8{\cdot}7\times10^{-8}$

From this it appears that we are not yet in a position to calculate more than the order of magnitude of the constant $\frac{4}{3}(\alpha/e)^2$ which the electronic theories give as the value of $k/\kappa t$.

In order to see how far the theoretical result agrees in other respects with the observed facts, the values of $k/\kappa t$ at various temperatures for the metals and alloys tested have been calculated from the values of the thermal and electric conductivities given in the tables on pp. 426 and 437. The results are shown in the following table and in the curves on Plate 31.

The corresponding values calculated from JÄGER and DIESSELHORST'S experiments at 18° C. and 100° C. are also given for comparison.

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* The mean of J. S. TOWNSEND'S ('Phil. Mag.,' 45, p. 133 (1898)), J. J. THOMSON'S ('Phil. Mag.,' V., p. 355 (1903)), and H. A. WILSON'S results ('Phil. Mag.,' V., p. 441 (1903)).

† KELVIN, 'Phil. Mag.,' 4, p. 197 (1902).

‡ KELVIN, 'Phil. Mag.,' 4, p. 301 (1902).

§ J. H. JEANS, 'Phil. Mag.,' 8, p. 694 (1904).

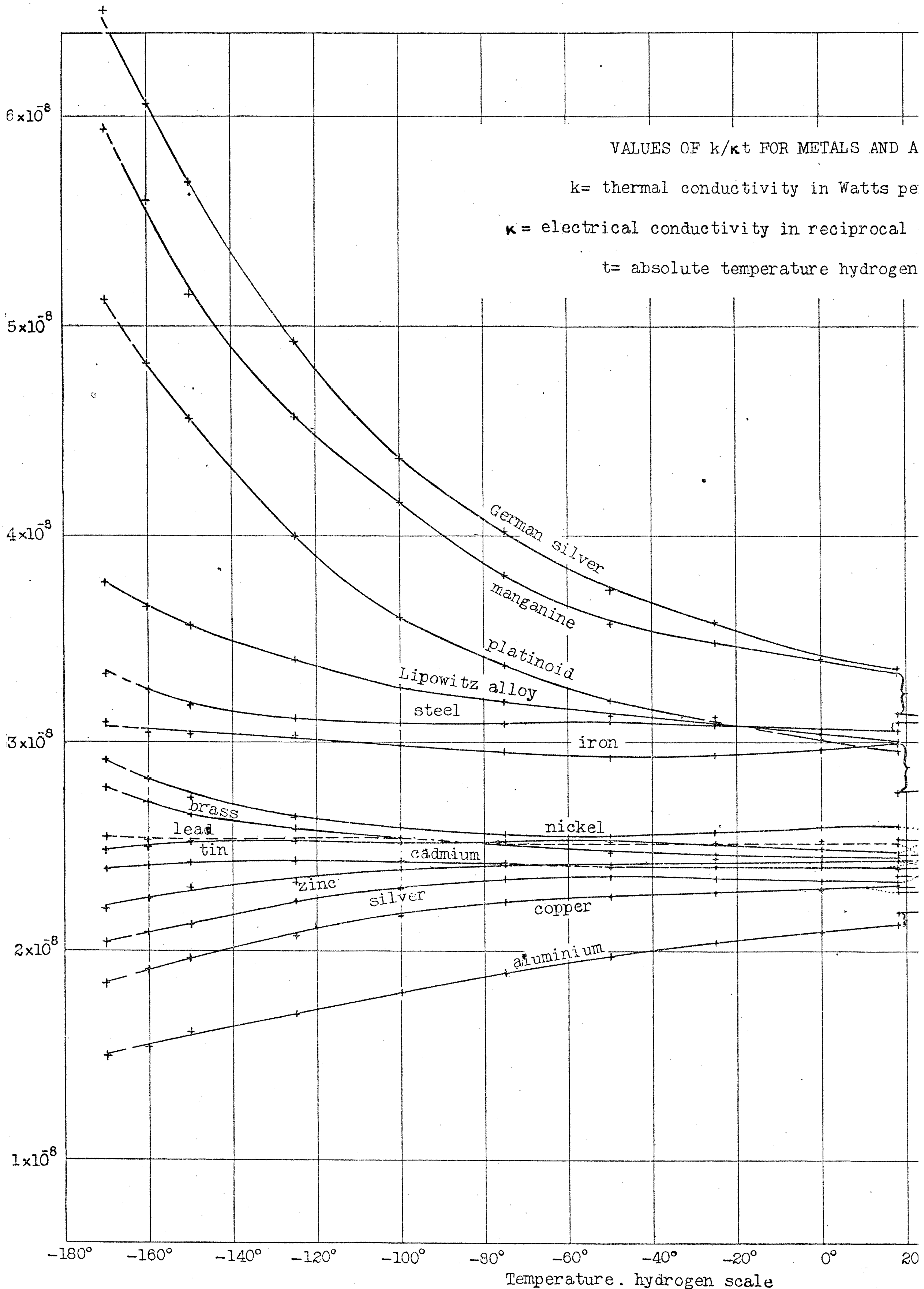
‖ M. PLANCK, 'Ann. der Phys.,' 4, p. 566 (1901).

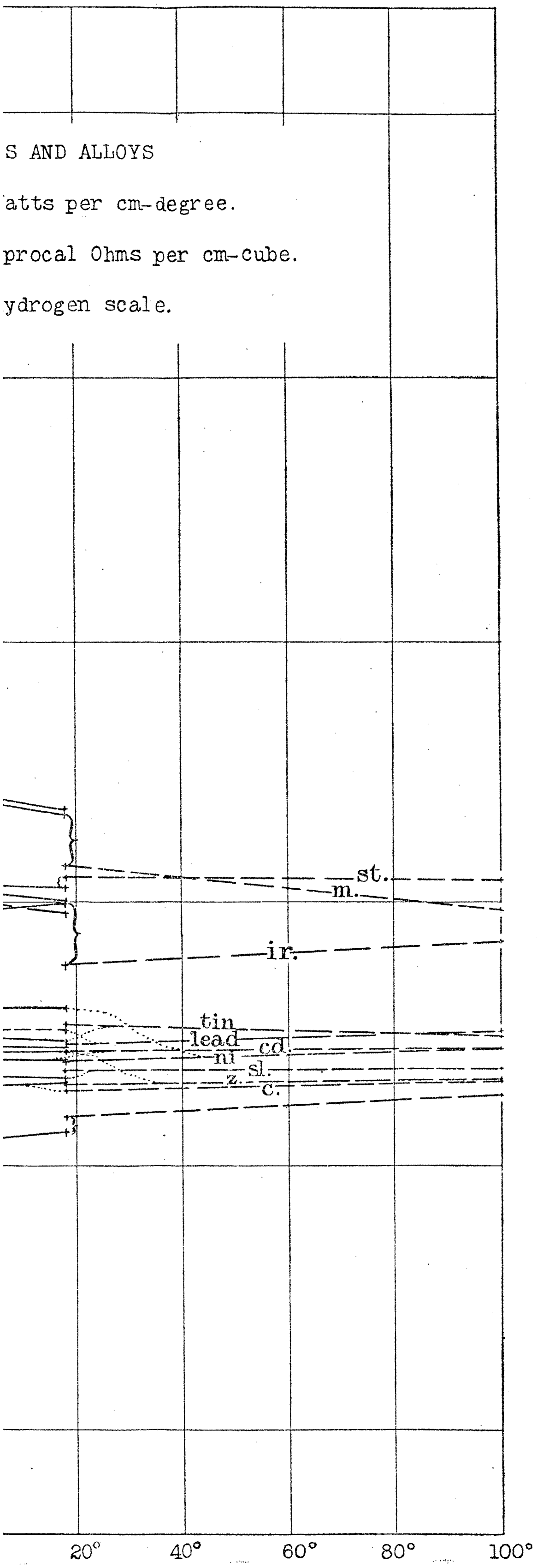
¶ H. A. LORENTZ, 'Versl. Akad. van Wet.,' 14, p. 518, &c. (1906).

** J. J. THOMSON, 'Phil. Mag.,' 5, p. 355 (1903).

†† H. A. WILSON, 'Phil. Mag.,' 5, p. 441 (1903).

‡‡ This line is added since Professor TOWNSEND'S proof that when a gas is ionised by Röntgen rays there are only half so many positive as negative ions produced ('Proc. Roy. Soc.,' 80, p. 210, 1908), seems to render it advisable to take into account the possibility of the H atom in electrolysis having a charge $2e$.


VALUES OF k/κt FOR METALS AND A
k= thermal conductivity in Watts pe
κ = electrical conductivity in reciprocal
t= absolute temperature hydrogen
6×10⁻⁸
5×10⁻⁸
4×10⁻⁸
3×10⁻⁸
2×10⁻⁸
1×10⁻⁸
German silver
manganine
platinoid
Lipowitz alloy
steel
iron
brass
lead
tin
nickel
cadmium
zinc
silver
copper
aluminium
−180°
−160°
−140°
−120°
−100°
−80°
−60°
−40°
−20°
0°
20
Temperature, hydrogen scale


S AND ALLOYS
atts per cm-degree.
procal Ohms per cm-cube.
ydrogen scale.
st.
m.
ir.
tin
lead
cd.
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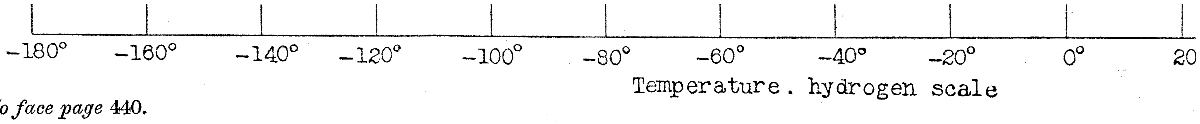
sl.
z.
c.
20°
40°
60°
80°
100°



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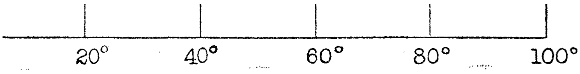

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE of Values of $k/\kappa t$ Calculated from the Present Experiments Compared with those Calculated from JÄGER and DIESSELHORST'S Results.

k, the thermal conductivity in watts per centimetre-degree.
κ, the electrical conductivity in reciprocal ohms per centimetre cube.
t, absolute temperature on the hydrogen scale.

	From the present experiments.										From JÄGER and DIESSELHORST'S experiments.	
Temperatures . . .	−170° C.	−160° C.	−150° C.	−125° C.	−100° C.	−75° C.	−50° C.	−25° C.	0° C.	+18° C.	18° C.	100° C.
Aluminium	1·50	1·54	1·61	1·70	1·81	1·90	1·98	2·04	2·09	2·13	2·19	2·27 × 10⁻⁸
Copper	1·85	1·92	1·97	2·07	2·17	2·23	2·26	2·28	2·30	2·32	2·29	2·32
Silver	2·04	2·09	2·13	2·24	2·29	2·34	2·36	2·35	2·33	2·33	2·36	2·37
Zinc	2·20	2·25	2·30	2·33	2·39	2·41	2·40	2·44	2·45	2·43	2·31	2·33
Cadmium	2·39	2·41	2·42	2·43	2·43	2·42	2·41	2·40	2·40	2·39	2·43	2·44
Tin	2·48	2·49	2·53	2·52	2·51	2·52	2·53	2·51	2·49	2·47	2·53	2·49
Lead*	2·55	2·53	2·52	2·54	2·54	2·51	2·52	2·51	2·53	2·51	2·46	2·51
Nickel	2·92	2·82	2·73	2·63	2·59	2·56	2·55	2·57	2·59	2·59	2·40	2·44
Iron	3·10	3·05	3·04	3·03	2·98	2·95	2·93	2·94	2·97	2·99	2·76	2·85
Steel	3·34	3·26	3·19	3·11	3·09	3·09	3·10	3·09	3·06	3·05	3·10	3·09
Brass	2·78	2·71	2·65	2·57	2·54	2·51	2·47	2·46	2·45	2·45		
Lipowitz alloy . .	3·78	3·66	3·57	3·40	3·27	3·20	3·13	3·11	3·05	3·00		
Platinoid	5·13	4·82	4·56	4·00	3·61	3·38	3·21	3·13	3·01	2·96		
Manganine	5·94	5·60	5·16	4·57	4·16	3·81	3·58	3·49	3·41	3·34	3·14	2·97
German silver . . .	6·51	6·06	5·69	4·93	4·37	4·02	3·75	3·59	3·41	3·36		

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* MACHIA, 'Acc. Lincei Atti,' 16, p. 507 (1907), finds a rather greater increase of the conductivity constant for lead at low temperatures.

The curves show that there is a tendency for the values of the conductivity constant $k/\kappa t$ to collect in the neighbourhood of the value $2 \cdot 4 \times 10^{-8}$, within the range given by the electronic theories, and that this tendency is rather more marked in the values calculated from JÄGER and DIESSELHORST'S results than in those calculated from the present experiments. Lead, tin, and possibly cadmium give values independent of temperature over the wide range covered, and therefore support the statement made by L. LORENZ in 1882. Zinc, silver, copper and aluminium show a marked decrease of the constant as the temperature decreases. Nickel, steel, and possibly iron, show an increase as the temperature decreases. This appears to be the characteristic behaviour of alloys. The values for aluminium and copper are decidedly below, while those for nickel and iron are decidedly above, the value 2·4. All the alloys give values above this, and, with the exception of brass, decidedly above. This large value of the constant for alloys especially at low temperatures is in keeping with and extends the results obtained previously by GRÜNEISEN,* who found that increase of impurity in copper and iron at ordinary temperatures increased the value of the constant.

It is necessary to bear this increase in mind in basing any argument on the values observed. The high values for nickel and iron may, since both contained a little impurity (about 1 per cent.), be ascribed to the impurity. In the cases of the other metals, with the exception of aluminium, which contained a little impurity (about 1 per cent.), it is not probable that any small amount of impurity present in them could have materially influenced the positions of the curves. In the case of aluminium it seems possible that the curve at low temperatures may be a little high, owing to the small quantity of impurity present in the metal.

The only conclusion which can be drawn from the curves seems to be that the Lorenz law of constancy of the value of $k/\kappa t$ at different temperatures for each metal has only a restricted application, and that although the metals which support the law give nearly the same value, 2·4, for the constant,† the other metals have values of the constant which differ from 2·4 by amounts far outside the limits of experimental error.‡

These facts are not of such a character as to provide strong support for any of the electronic theories mentioned on p. 439 in their present forms.

A modified theory has been put forward recently by Professor J. J. THOMSON§ to get over a difficulty he pointed out in the older theories, which made the energy

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* E. GRÜNEISEN, 'Ann. der Phys.,' 3, p. 43 (1900).

† It is worthy of note that for mixtures of these metals, MATTHIESSEN, 'Ann. der Phys.,' 110, p. 190 (1860), found that the electrical conductivities, and SCHULZE, 'Ann. der Phys.,' 9, p. 555 (1902), that the thermal conductivities, could be calculated from the relative volumes of the constituents present.

[‡ *Note added June* 1.—The variations of the values of $k/\kappa t$ for the same metals at higher temperatures will form the subject of a future communication.]

§ J. J. THOMSON, 'Journ. Inst. Elect. Eng.,' 38, p. 455 (1907), or 'Corpuscular Theory of Matter,' p. 86.

absorbed by the electrons in 1 gramme of a metal during an increase of temperature of 1° C. far exceed the observed specific heat of the metal. In this theory the electrons pass from one doublet, consisting of a positively and negatively charged atom, to another similar doublet, which have arranged themselves along the line of electric force, as in the old theory of electrolysis the molecules of the Grotthus chains were supposed to do. If d is the distance apart of the charges of the doublet, and b is the distance between the doublets, THOMSON shows that $k/\kappa t$ is $9b/8a$ times the value found on the former theory. Since b/a may vary in value from metal to metal, and in the same metal at different temperatures, this theory may by a proper choice of the laws of variation be brought into unison with the observed facts.

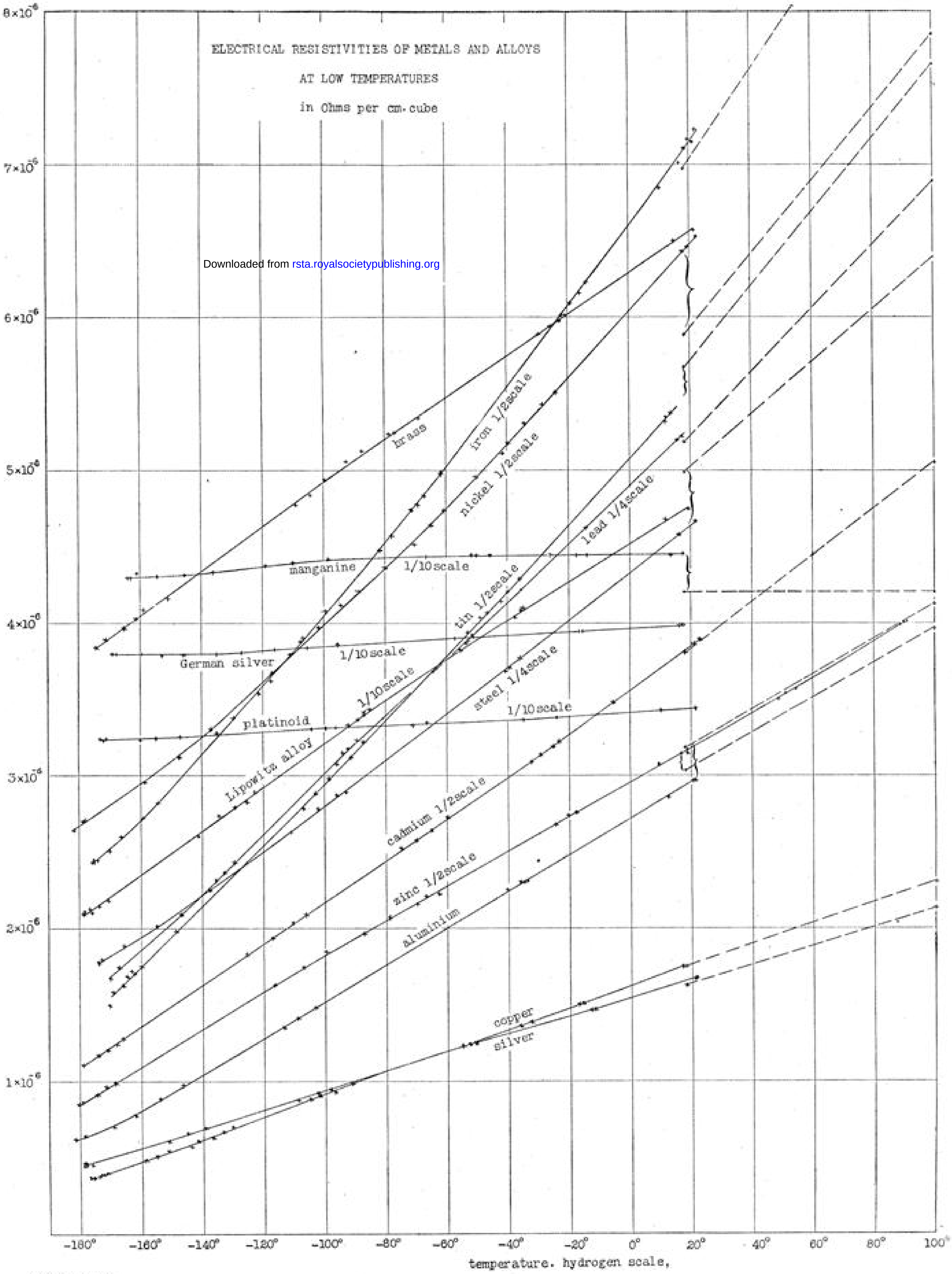
It is, however, open to doubt whether any theory which does not take into account the atomic differences of the various metals can give a satisfactory explanation of the whole facts. The nearness to each other of the curves for metals belonging to the same chemical group, Plate 31, suggests that atomic properties play their part in determining the behaviour of a metal to the flow of heat and electricity through it.

Electronic theories have regarded the free electrons as the sole carriers of energy from molecule to molecule, and have therefore been unable to deal with the thermal conductivities of electrical insulators, in which there can be no free electrons. Now the thermal conductivities of some substances which insulate electrically are as large as those of the worse conducting metals, *e.g.*, the thermal conductivity of quartz along the axis is 0·029, while that of bismuth is only 0·016 to 0·019. It seems, therefore, that a theory of heat conduction which ascribes it to free electrons alone must be a very partial account of it.

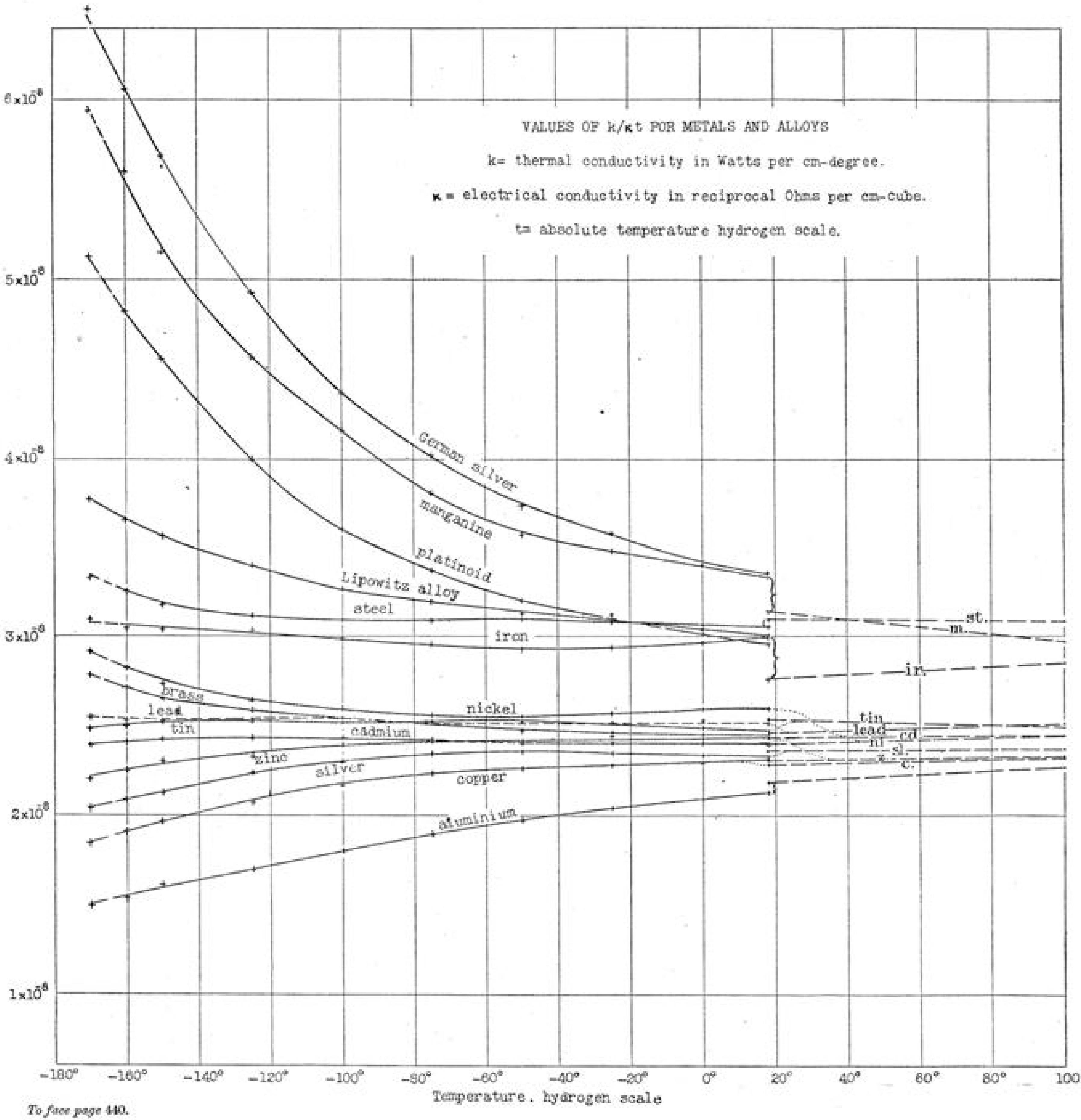
The fact pointed out on p. 427, that the variation of the thermal conductivities of the metals with temperature is of the same character as, though less marked than, the corresponding variations for electrical insulators, and that alloys vary in a manner similar to the only mixture (glass) of insulators yet investigated, both support this contention, that theories which depend solely on free electrons as carriers can hardly claim to be adequate expressions of the phenomena.

This investigation was commenced in Manchester, where a liquid air plant was available. I have to thank Dr. E. C. C. BAILY and the authorities of University College for kindly providing me with liquid air in London.

I have also to thank the Committee administering the Government Grant for Scientific Investigations for a large proportion of the funds out of which the materials used in the investigation have been purchased.



ELECTRICAL RESISTIVITIES OF METALS AND ALLOYS AT LOW TEMPERATURES in Ohms per cm. cube



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